# Strategic Information Platforms in Transportation Networks

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***Abstract*— We investigate the effect of information/navigation platforms in transportation networks. Specifically, we analyze the outcome when these platforms are owned by for-profit strategic companies such as Google and Apple. We consider two business models, one that makes a profit through advertisements and user information collection, and one that generates revenue from its user by charging a subscription fee. We show that social welfare in an environment with a single platform can be higher than the one when multiple platforms compete with one another. This is in contrast to the standard result for classical goods where competition always improves social welfare. Most importantly, we show that in a competitive environment with multiple platforms, each platform finds it optimal to disclose its information perfectly about the current condition of the network for free. Consequently, we show that in a competitive market most information platforms must have an ad-based business model and reveal perfect information about the transportation network. Our results provide a purely economic justification on why in practice no navigation application attempts to utilize its superior information to improve the congestion by disclosing partial information as suggested previously in the literature.**

## I. Introduction

GPS-enabled devices and navigation applications such as Google Maps and Waze enable drivers to make a more informed routing decision by providing real-time traffic data and routing recommendations. It has been projected that by 2020, almost 72% of the U.S. population will own a smart phone, which corresponds to 93% of all mobile phones [1]. It has been estimated that %77 of smart phone users regularly use navigation apps, where Google Maps (67%), Waze (12%), and Apple Maps (11%) are the most popular apps among them [2]. Similarly, it has been estimated that the market size of in-dash navigation systems for cars grows at 12.47% annually between 2017-2022 [3]. With the wide-spread use of these new technologies, there is an increasing interest to study the effect of them on the induced traffic in transportation networks.

Several theoretical works have investigated the effect of providing information to drivers [4]–[9]. They have shown that the consequences of the wide-spread use of these technologies are ambiguous. They identified instances where providing information to drivers result in social welfare loss and an increase in the overall traffic congestion. These results have been supported by a few empirical evidences on the negative effect of these technologies on traffic congestion [10]–[13].

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Motivated by these results, several studies have investigated the problem of information disclosure in transportation networks [14]–[18]. Using the Bayesian persuasion [19] and information design [20] frameworks, these studies have investigated the problem of information mechanism design so as to reduce the overall traffic congestion and improve social welfare. They show that it is not always socially optimal to disclose perfectly all information to drivers. In contrast, in most environments, the optimal mechanism is to disclose partial information to drivers and use the routing recommendations as a coordination device between drivers' routing decisions [14], [21]. These studies assume that the information platform is benevolent and design its information mechanism, *i.e.*, routing recommendation policy, to maximize the social welfare or minimize the overall traffic.

However, major navigation applications such as Google Maps or Apple Maps are owned by for-profit companies. It is hard to justify that their objective is to improve social welfare rather than their own private profit. We argue that although these studies demonstrate the potential positive impact an information platform may have on transportation networks, they do not adequately explain the decisions made in practice by major navigation applications/information platforms operated by for-profit companies such as Google or Apple.

In this paper, we study a problem where information platforms are designed by strategic entities that have private objectives. We consider two business models for an information platform. In the first business model, which we refer to as *ad-based model*, each platform's objective is to maximize its number of users. This business model corresponds to the several popular navigation applications which generate revenues either from advertisement or information gathering from users (*e.g.*, Waze, Google Maps). We also consider a second business model where a platform charges its users a subscription fee for providing information (*e.g.*, TomTom and Locus Map); we note that currently almost all fee-based services are premium services offered as an upgrade option either for in-dash navigation systems (*e.g.*, live traffic in TomTom) or as an in-app purchase that targets a very specific market segment and demographic (*e.g.*, routing preferences for biking and hiking in Locus Map).

We first consider a scenario with a single platform. In an ad-based business model, we argue that the platform's objective is to maximize the gap between the utility of a user and that of someone who decides not to use the platform. Consequently, we show that the optimal policy for the platform can be different from the socially efficient policy. With a single fee-based platform, we show that it can be optimal for the platform to set its price such that only a fraction of drivers join the platforms. For both business models, we provide conditions under which the socially optimal policy is also optimal for a single strategic platform.

Next, we consider a scenario with multiple information platforms competing over users. We show that the competition among platforms pushes all of them to reveal information perfectly to their users. Moreover, if a platform has a fee-based business model, it is compelled to set its price to zero at equilibrium and cannot make a positive profit. Therefore, we show that in a competitive market, the only business model that can make a profit in long-run is the ad-based business model. Additionally, we show that all platforms must disclose their information completely and no partial information policy is sustainable as an equilibrium. Our results reflect the current market outcome observed in practice, where all major navigation apps are free, and provide the best routing recommendation to every user given their information. We also show through a numerical example that the competition among strategic platforms can result in lower social welfare compared to that under a single platform. This deviates from the classical result for a standard good, where competition improves social welfare. The result can be explained by the negative externality created by revelation of additional information in a competitive environment.

The rest of the paper is organized as follows. In Section II, we present the model. In Section III, we provide a brief overview of the preliminaries necessary to set up the problem. In Section IV, we lay out the information design framework. In Section V, we investigate the outcome when there exists a single platform. In Section VI, we analyze the outcome when multiple information platforms exist. In Section VII, we demonstrate our results through a numerical example and provide an instance where social welfare is lower under multiple platforms than that under a single platform. We conclude in Section VIII. All proofs are omitted due to space limitation and can be found online in [22].

## II. Model

Consider a parallel two-link network as in Figure 1. There is a unit mass of non-atomic agents/drivres who want to travel from node $O$ to node $D$ on one of the two routes. The travel time through each route is determined by the condition of that route as well as the traffic congestion on it. Let $a$ denote the condition of the first route, called *safe route* $S$, where $a$ is a positive constant. The condition of the second route, called *risky route* $R$, is a random variable $\theta$ that takes values in $\{\theta_1, \ldots, \theta_N\}$ with probability $\{p_1, \ldots, p_N\}$, where $\theta^1 < \theta^2 < \ldots < \theta^N$ and $\sum_{i=1}^{n} p_i = 1$. Let $f^s$ and $f^r$ denote the mass of drivers that travel through the safe and risky routes, respectively, where $f^S + f^R = 1$. If a driver takes route $S$ his utility is given by

$$u^S = a - f^S. \quad (1)$$

Similarly, if a driver takes route $R$ his utility is given by

$$u^R = \theta - f^R. \quad (2)$$

Each driver chooses a route trying to maximize his (expected) utility. We assume that the value $a$ of the safe route $S$ is fixed and known to all drivers. However, the value $\theta$ of the risky route $S$ is random and not known to drivers. We assume that the prior common probability mass function $\{p_1, \ldots, p_N\}$ is known to all drivers. Each driver can gain additional information by subscribing to a platform that directly or indirectly discloses information about the realization of $\theta$.

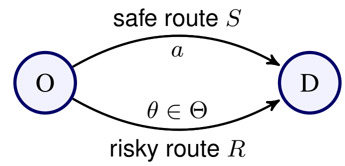

Fig. 1: The two-link network

## III. Preliminaries

Before we proceed to the analysis of outcomes under strategic information/recommendation platforms, we briefly discuss three benchmark cases: no information disclosure, full information disclosure, and socially efficient information disclosure. These benchmarks are developed based on results appearing in [23].

### A. No information disclosure

Consider a case where there exists no information platform. In this case, drivers choose their routing decisions based on a common prior belief about $\theta$. Define $\Delta := a - \mathbb{E}\{\theta\}$ as the prior expected difference between the routes' conditions. Let $x_{[0,1]}$ denote the projection of $x$ onto the interval $[0,1]$ given by $\min\{\max\{x, 0\}, 1\}$. It is easy to show that there exists a unique equilibrium where

$$f^{S,\text{no info}} = \left[\frac{1}{2} + \frac{1}{2}\Delta\right]_{[0,1]}, f^{R,\text{no info}} = \left[\frac{1}{2} - \frac{1}{2}\Delta\right]_{[0,1]}. \quad (3)$$

The expected social welfare $W$, *i.e.* the total utility of all drivers, is given by

$$W^{\text{no info}} = \frac{a+\mu-1}{2} + \max\{\frac{|\Delta|-1}{2}, 0\}.. \quad (4)$$

### B. Full information disclosure

Alternatively, consider a case where an information platform reveals to all drivers the realization of $\theta$ perfectly. Then, drivers choose their routing decisions based on the realization of $\theta$. It is easy to show that there exists a unique equilibrium outcome under full information where

$$f^{S,\text{full info}}(\theta) = \left[\frac{1}{2} + \frac{1}{2}\Delta_\theta\right]_{[0,1]}, f^{R,\text{full info}}(\theta) = \left[\frac{1}{2} - \frac{1}{2}\Delta_\theta\right]_{[0,1]}. \quad (5)$$

where $\Delta_\theta := a - \theta$. The social welfare under full information scenario is given by[2]

$$W^{\text{full info}} = \frac{a+\mu-1}{2} + \mathbb{E}\left\{\max\{\frac{|\Delta_\theta|-1}{2}, 0\}\right\}. \quad (6)$$

[2]In our linear model, if $|\Delta_\theta| \leq 1$ for all $\theta$, the expected social welfare under full information case, given by (6), is identical to the one under no information case, given by (4). However, in general, the social welfare under no information and under full information are not identical; the difference between the social welfare under these two regimes could be either positive, zero, or negative depending on the network topology, functional form of utility functions, and probability distribution of $\theta$; see [8], [14].

### C. Socially Efficient Outcome

It is well known that the equilibrium outcome in congestion games is socially inefficient [24]. For the scenario considered here, the socially efficient outcome is given by

$$f^s(\theta) = \left[\frac{1}{2} + \frac{\Delta_\theta}{4}\right]_{[0,1]}, f^r(\theta) = \left[\frac{1}{2} - \frac{\Delta_\theta}{4}\right]_{[0,1]}. \quad (7)$$

Comparing (7) to the routing outcomes (3)under the full information case, in the socially optimal outcome the difference between the traffic on the two routes is given by $\frac{\Delta_\theta}{2}$ instead of $\Delta_\theta$; that is, mindful of congestion externality, the social planner does not over-utilize the better route to the point where there is no difference between the net utility of the two routes.

Under (7), the optimal social welfare is given by

$$\begin{aligned} W^{\text{efficient}} =& \frac{a+\mu-1}{2} + \frac{\Delta^2}{8} + \frac{\sigma^2}{8} \\ &+ \mathbb{E}\left\{ \mathbb{1}_{\{|\Delta_\theta|>1\}} \left[ \frac{|\Delta_\theta|-1}{2} - \frac{\Delta_\theta^2}{8} \right] \right\}, \quad (8) \end{aligned}$$

where $\mathbb{1}_{\{|\Delta_\theta|>1\}}$ denotes the indicator function for event $\{|\Delta_\theta| > 1\}$, *i.e.* when the effect of one route's condition is large enough that it results in a greater net utility compared to the other one even when chosen by all drivers.

## IV. Information Design

An information platform/navigation application with the knowledge of $\theta$ can disclose a variety of informative signals to the drivers that include the full information and no information scenarios discussed above as special cases. In general, an information platform has to specify (i) the communication alphabet it uses to send informative signals to every driver, and (ii) the information policy that for every realization of $\theta$ determines the probability of sending a set of joint signals to all drivers.

**Remark 1.** *Throughout the paper, we only consider the class of private information mechanisms and do not explicitly consider public information mechanisms. We say an information mechanism is private if the platform can disclose different information (recommendation) to different drivers; similarly, an information mechanism is public if all drivers receive the same information. We note that the set of public information mechanisms is a subset of private information mechanisms. We conjecture that our main results still hold even if we restrict attention to public information mechanisms.*

### A. Revelation Principle

We utilize the revelation principle for information design [20], and restrict attention, without loss of generality, to a class of *recommendation policy* that satisfies a set of *obedience constraints* for all drivers. We say an information disclosure mechanism is a recommendation policy if the information platform directly recommend a route to each driver for every realization of $\theta$. That is, the alphabet the platform uses to communicate with every driver is the same as the action set for that driver. We say a recommendation policy satisfies the obedience constraints if for every driver, it is a best response to take the route recommended to him.

We note that when there are multiple mechanism designers in a market competing with each other, the standard revelation principle does not hold in general [25]. This is because with multiple principals there exists a richer set of mechanisms for each principal in which mechanisms can potentially depend on the market information they gather from the agents. This may contain information about competing mechanisms, and enables each principal to choose a more complex mechanism that utilizes information dependent on other principals' mechanisms. For instance, the well-known *"matching price mechanism"*, where a buyer sets a price with a guarantee to match any competitor's lower price is an example of such mechanisms, which cannot be captured by the standard revelation principle. To keep our analysis tractable and avoid such difficulties, we restrict attention to mechanisms where an information platform cannot offer a contingent mechanism where its outcomes depend on other competing information mechanisms. This is consistent with how currently information platforms operate in practice. Therefore, we follow the standard revelation principle to formulate the problem for each information platform.

### B. Obedience Constraints

Following an argument similar to that in [14], a recommendation policy is characterized as $x : \{\theta^1, \dots, \theta^N\} \to [0,1]$ where $x(\theta^i)$ (resp. $1-x(\theta^i)$) determines the probability that a driver receives a recommendation to choose route $S$ (resp. route $R$) when $\theta = \theta^i$. We note that, given a realization of $\theta$, the probability of recommending each route to every driver is an independent and identically distributed Bernouli random variable with parameter $x(\theta)$.

Consider a simple case where there is only a single information platform with all drivers subscribed to it. Given a recommendation policy $x(\theta)$, the set of obedience constraints can be written as follows. Consider a case where a driver receives a recommendation to choose route $S$. A driver's ex-post belief about $\theta = \theta^i$ is given by $\mathbb{P}\{\theta = \theta^i | \textit{recommendation}\} = \frac{p(\theta^i)x(\theta^i)}{\sum_{j=1}^N p(\theta^j)x(\theta^j)}$. Therefore, it is a best response for him to follow the recommendation and choose route $S$ if and only if

$$\begin{gathered} \sum_{i=1}^N \frac{p(\theta^i)x(\theta^i)}{\sum_{j=1}^N p(\theta^j)x(\theta^j)}(a - x(\theta^i)) \\ \geq \\ \sum_{i=1}^N \frac{p(\theta^i)x(\theta^i)}{\sum_{j=1}^N p(\theta^j)x(\theta^j)}(\theta - (1 - x(\theta^i))), \quad (9) \end{gathered}$$

where $\frac{p(\theta^i)x(\theta^i)}{\sum_{j=1}^N p(\theta^j)x(\theta^j)}$ denote the driver's ex-post belief that $\theta = \theta^i$. The left-hand side of (9) denotes the driver's expected utility if he follows the recommendation, and right-hand side of (9) captures his expected utility if he deviates and choose route $R$.

Similarly, if a driver receive a recommendation to choose route $R$, it is a best response for him to follow the recommendation if and only if

$$\sum_{i=1}^{N} \frac{p(\theta^i)(1-x(\theta^i))}{\sum_{j=1}^{N} p(\theta^j)(1-x(\theta^j))}(\theta-(1-x(\theta^i))) \geq \sum_{i=1}^{N} \frac{p(\theta^i)(1-x(\theta^i))}{\sum_{j=1}^{N} p(\theta^j)(1-x(\theta^j))}(a-x(\theta^i)). \qquad (10)$$

The set of obedience constraints for a case with single platform where all drivers (with unit mass) are subscribed to it are given by inequalities (9) and (10). The set of obedience constraints for scenarios with multiple information platforms or when not all drivers are subscribed to a platform can be written similarly; we omit the explicit form of these constraint due to space limitation.

### C. Social Planner

Our main objective in this paper is to study the market outcome in the presence of strategic information platforms. The following result is from [14] and concerns an environment with a single information platform operated by a social planner that is a non-strategic and social-welfare maximizer. We use this result as a benchmark to demonstrate the market inefficiency because of the profit-seeking behavior of strategic information platforms.

**Theorem 1.** *The socially efficient outcome $x^{\text{efficient}}$ is implementable through an information mechanism if and only if*

$$\sigma^2 \geq 2|\Delta| - \Delta^2. \qquad (11)$$

The result of Theorem 1 can be interpreted as follows. If in expectation the two routes are identical, *i.e.* $|\Delta| = 0$, a social planner can always achieve the efficient outcome. However as $|\Delta|$ increases, *i.e.*, one route is increasingly better in expectation, drivers start to develop a preference for the better route. Therefore, to persuade drivers to change their behavior the social planner need to possess a higher *informational power*, measured as $\sigma^2$, to achieve the efficient routing outcome.

## V. Single Platform

Consider a monopolistic market where a strategic information platform provides recommendation services. We investigate two possible business models for the platform. In the first business model, users can use the platform service for free and the platform collects revenue through targeted advertisement (*e.g.*, Waze, Yelp) and/or information gathering (*i.e.*, Google Maps, Waze). In the second business model, the platform sets a subscription fee and collects revenue by charging its users. In both scenarios, each driver individually decides to either join the platform or opt out based on his expected net utility.

Let $m$ denote the population size of platform users, $m \leq 1$. Based on the framework presented in Section IV, let $x(\theta)$ denote the fraction of the platform's users that receive the recommendation to take route $S$. Let $y$ denote the fraction of the uninformed drivers that take route $S$. Conditioned on the realization of $\theta$, define the utility of a driver taking the safe and risky routes as

$$u^s(\theta) = a - mx(\theta) - (1-m)y, \qquad (12)$$

$$u^r(\theta) = \theta - m(1-x(\theta)) - (1-m)(1-y), \qquad (13)$$

respectively. Therefore, the expected utility of uninformed drivers and the platform's users are given by

$$U_0 := \max\{\mathbb{E}\{u^s(\theta)\}, \mathbb{E}\{u^r(\theta)\}, \qquad (14)$$

$$U_1 := \mathbb{E}\{x(\theta)u^s(\theta) + (1-x(\theta))u^r(\theta)\}. \qquad (15)$$

In what follows, we investigate the possible outcomes when there exists a single information platform operating based on the aforementioned business models.

### A. Ad-based platform

The ad-based business model captures existing services like Waze. Assuming an average profit per each user, the platform's revenue is proportional to the size of its user base given by $\alpha m$. Therefore, the platform's objective is to maximize the population of its users $m$.

Each driver decides to use the platform if and only if the expected utility from using the platform is greater than or equal to that from not using it. Notice that the expected utility of an uninformed driver also depends on the recommendation policy $x(\theta)$ chosen by the platform. This is different from the standard mechanism design framework where an agent's outside option/reservation utility is assumed to be fixed.

For example, consider the case where the probability distribution of $\theta$ is binary with $p_i = 0.5$ where $\theta \in \{1.25, 3.25\}$, and $a = 2$. Define recommendation policy $x^\alpha(\theta)$ as

$$x^\alpha(\theta) = \left[\frac{1}{2} + \frac{\Delta_\theta}{\alpha}\right]_{[0,1]}.$$

Let $U_0^\alpha$ and $U_1^\alpha$ denote the expected utility of an uninformed driver and an informed driver at equilibrium for recommendation policy $x^\alpha(\theta)$. It is easy to verify that $x^\alpha$ satisfies the obedience constraints for $\alpha \leq 8.5$. As seen in Figure 2, both $U_0^\alpha$ and $U_1^\alpha$ change as $\alpha$ varies. We note that for $m = 1$, policy $x^\alpha(\theta)$ is the same as full information outcome for $\alpha = 2$ and is equal to the socially optimal outcome for $\alpha = 4$; see (5) and (7). Therefore, as $\alpha$ increases from 2 to 4, $U_1^\alpha$ (*i.e.* the social welfare) increases. For $\alpha \in [4, 8.5]$, $U_1^\alpha$ decreases as the platform under-utilizes the better route for each realization of $\theta$. As we noted before, $U_0^\alpha$ endogenously varies as we change $\alpha$. In the above example, the best outside option is to choose route $R$ for all $2 \leq \alpha \leq 8.5$. For $2 \leq \alpha \leq 2.5$ the platform recommends to *all* drivers to take route $R$ when $\theta = 3.25$ and it makes mixed recommendation (with prob. $0.5 - \frac{0.75}{\alpha}$) when $\theta = 1.25$. Therefore, as $\alpha$ increases from 2 to 2.5 the expected congestion for route $R$ increases, and thus, $U_0^\alpha$ decreases. For $\alpha > 2.5$ the platform makes mixed recommendations for both realizations of $\theta$; as $\alpha$ increases the mixing rate increases faster for $\theta = 3.25$. Therefore, the expected congestion for route $R$ decreases,

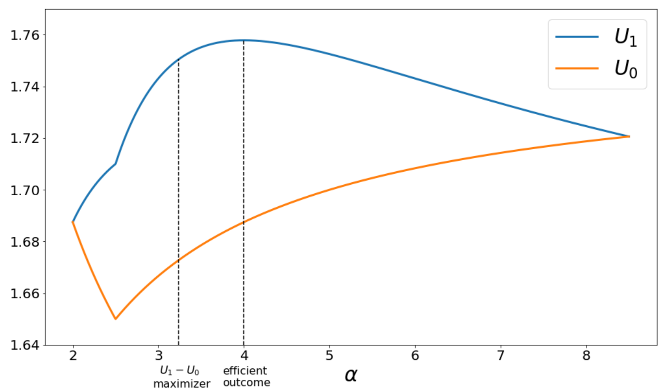

Fig. 2: $U_0^\alpha$ and $U_1^\alpha$ when $\theta \in \{1.25, 3.25\}$, each with probability $0.5$, $a = 2$, and $m = 1$.

and thus, $U_0^\alpha$ increases. From Figure 2, it is clear that the maximum value of $U_1 - U_0$ and $U_1$ do not coincide.

The example above demonstrates the additional complexity present in designing a recommendation policy because of the endogenous dependence of the outside option $U_0^\alpha$ on recommendation policy $x(\theta)$. We note that for all $\alpha \in (2, 8.5)$ all drivers strictly prefer to join the platforms. The next result shows that this is not a surprising observation as the drivers' preference to join the platform is directly connected to the obedience constraints.

**Lemma 1.** *For any recommendation policy that satisfies the obedience constraints, all drivers weakly prefer to join the ad-based platform, that is, $m = 1$. They strictly prefer to join the platform if and only if all obedience constraints are satisfied strictly.*

An immediate corollary of Lemma 1 is that for every recommendation policy that satisfies the obedience constraints, we have $U_1 \geq U_0$. Consequently, within the context of problem formulation we consider so far, the platform does not have a unique optimal recommendation policy. Nevertheless, in real-world applications, there are additional considerations that may affect the platform's optimal choice. One such consideration is the fact that a driver's decision to join the platform can be insensitive to a small utility gain from using the platform. In this case, the platform may want to maximize the difference between the expected utility when a driver joins the platform and that when he opts out. In this case, the optimal policy is given by

$$\max_{x(\theta)} \quad U_1 - U_0 \tag{16}$$

subject to

$$m = 1, x(\theta) \in [0, 1]$$
$$(12) - (15),$$
$$\text{obedience constraints for } x(\theta).$$

We note that the maximum gap between $U_1^\alpha$ and $U_0^\alpha$ does not occur at the social maximizing recommendation policy in general; see the example above where $U_1 - U_0$ is maximized at $\alpha - 3.25$ while $U_1$ (social welfare) is maximized at $\alpha = 4$.

Due to the endogenous dependency of $U_0$ on $x(\theta)$, it is not straightforward to provide a closed-form solution to the optimization problem (16). Nevertheless, we can provide conditions under which the optimal policy for a single ad-based platform coincides with the socially optimal outcome.

**Theorem 2.** *Consider the optimization problem (16) and assume that probability distribution of $\theta - a$ is symmetric around $0$. Then, the optimal policy is given by*

$$x(\theta) = \left[\frac{1}{2} + \frac{\Delta_\theta}{4}\right]_{[0,1]}, \tag{17}$$

*which is the same as the socially optimal outcome.*

The result of Theorem 2 states that there are conditions where a strategic platform find it optimal to implement a socially optimal recommendation policy. However, as we demonstrated above, this is not necessarily the case when the distribution of $\theta - a$ is not symmetric around $0$. We provide an additional numerical example in Section VII, where the platform's optimal policy is different from the socially optimal outcome.

### B. Fee-based platform

The second business model for a recommendation platform is the one where it generates revenue by charging its users a subscription fee $p$ for providing information like TomTom. The platform's objective in this case is to maximize $m(p)p$ where $m(p)$ denotes the size of users who choose to subscribe to the platform at price $p$. A driver decides to subscribe to the platform if the utility gain he gets by joining the platform is greater than or equal to the subscription fee $p$ he pays, *i.e.*,

$$U_1 - p \geq U_0. \tag{18}$$

Assuming that recommendation policy $x(\theta)$ satisfies the obedience constraints, the left-hand side of (18) denotes the utility of a driver if he joins the platform. The right-hand side captures his expected utility if he opts out and chooses one of the routes without receiving any information from the platform.

For drivers that opt out, they choose between routes $S$ and $R$ anticipating the congestion created by all drivers. Let $y$ denote the probability that an uninformed driver chooses route $S$.[3] At the equilibrium, we have

$$y = \left[\frac{1}{2} + \frac{\Delta - 2m\mathbb{E}\{x(\theta)\}}{2(1-m)}\right]_{[0,1]}, \tag{19}$$

given recommendation policy $x(\theta)$ that satisfies the obedience constraints. Therefore, in the fee-based business model, the platform maximizes its revenue by solving the following optimization problem:

$$\max_{p,m,x(\theta)} \quad pm \tag{20}$$

subject to

$$m \in [0, 1], x(\theta) \in [0, 1],$$
$$(12) - (15), (18), (19),$$
$$\text{obedience constraints for } x(\theta).$$

[3] We restrict attention to symmetric strategies for uninformed drivers.

We note that since the platform maximizes $pm(p)$, equation (18) is satisfied as an equality at an optimal solution. Therefore, one can write $p$ in terms of $m$ as

$$p(m) := U_1 - U_0 \tag{21}$$

Given $y$ and $m$, the platform chooses $x(\theta)$ to maximize the utility gain a driver realizes by joining the platform. Because of the nonlinear projection to $[0,1]$ in (19) (and a similar projection for $x(\theta)$) it is not straight-forward to find the closed-form solutions to (20) for a general probability distribution of $\theta$. Nevertheless, we can determine the solution to (20) when the distribution of $\theta - a$ is symmetric around 0.

**Theorem 3.** *Consider the optimization problem (20) and assume that probability distribution of $\theta - a$ is symmetric around 0. Then,*

*(i) for a fixed $m$, the optimal recommendation policy is given by*

$$x(\theta) = \left[\frac{1}{2} + \frac{\Delta_\theta}{4m}\right]_{[0,1]}, \tag{22}$$

$$y = \frac{1}{2}; \tag{23}$$

*(ii) the platform revenue $p(m)m$ is maximized at $m^* \in [\underline{m}, 1]$ where $\underline{m} = \min\{1, \max_i \frac{|\Delta_{\theta_i}|}{2}\}$.*

We can interpret the result of Theorem 3 as follows. Under the symmetry assumption for the distribution of $\theta - a$, part (i) states that maximizing the gap between $U_1$ and $U_0$ is equivalent to maximizing the welfare of platform users.[4] The result of part (ii) states that the platform maximizes its profit for a range of user sizes. As the number of platform users increases, the gain each user enjoys decreases due to the negative externality other drivers create. Therefore, $p(m)$ is a decreasing function of $m$. When $m$ is above $\underline{m}$, $p(m) \propto \frac{1}{m}$. Therefore, the platform is indifferent between user size $m^* \in [\underline{m}, 1]$. However, for $m < \underline{m}$ the platform find it profitable to grow its user size since the negative externality created by additional users, and thus, the reduction in $p(m)$, are not as high as to fully neutralize any profit gain.

The result of Theorem 3 provides conditions under which the socially optimal outcome is an optimal policy for a single fee-based platform. However, this is not necessarily the case when the distribution of $\theta - a$ is not symmetric. In general, the platform's optimal policy is different from the socially optimal outcome. That is, the platform find it optimal to set its subscription fee $p$ at a level where only a subset of users joins the platform, *i.e.* $m < 1$. We provide such an example in Section VII.

## VI. Platform Competition

In this section, we analyze the market outcome when there are multiple information platforms competing with each other. We assume that all information platforms have identical information about $\theta$. For ease of exposition, we first describe the model for the case when there are two platforms; however, our results hold for a general number of platforms.

Let $m_1$ and $m_2$ denotes the size of users for platform 1 and platform 2, where $m_1 + m_2 \le 1$. For now, we assume that each driver subscribes to at most one of the platforms. We later show that our results do not change even if we relax this assumption. Let $x_1(\theta)$, $x_2(\theta)$ and $y$ denote the fraction of drivers served by platform 1, platform 2, and uninformed drivers, respectively, that choose the safe route $S$. Given $x_1(\theta)$, $x_2(\theta)$ and $y$, the utility of the safe and risky routes are given by

$$u^s(\theta) = a - m_1 x_1(\theta) - m_2 x_2 - (1 - m_1 - m_2)y \tag{24}$$

$$u^r(\theta) = \theta - m_1(1 - x_1(\theta)) - m_2(1 - x_2(\theta)) - (1 - m_1 - m_2)(1 - y), \tag{25}$$

for every realization of $\theta$. The expected utility of an informed driver is then given by

$$U_0 := \mathbb{E}\{y u^s(\theta) + (1 - y) u^r(\theta)\}.$$

Similarly, let

$$U_1 := \mathbb{E}\{x_1(\theta) u^s(\theta) + (1 - x_1(\theta)) u^r(\theta)\},$$
$$U_2 := \mathbb{E}\{x_2(\theta) u^s(\theta) + (1 - x_2(\theta)) u^r(\theta)\}.$$

denote the expected utility of a driver subscribed to platforms 1 and 2.

**Remark 2.** *Throughout this paper, we say a recommendation policy chosen by a platform is a full/perfect information closure policy if the resulting equilibrium outcome is the same as if the platform discloses $\theta$ perfectly to its users.*

In the following, we investigate how market outcomes are affected by platform competition.

### A. Ad-based platforms

Consider a duopoly market where both platforms generate revenues through advertisement, and thus, they both aim to maximize the number of their users. The market equilibrium constraints, when both platforms are ad-based, are given by

$$\begin{aligned} U_0 &\ge \max\{U_1, U_2\}, && \text{if } m_1 + m_2 < 1, \\ U_1 &\ge \max\{U_0, U_2\}, && \text{if } m_1 > 0, \\ U_2 &\ge \max\{U_0, U_1\}, && \text{if } m_2 > 0. \end{aligned}$$

The following theorem characterizes the equilibrium outcome under competition with ad-based business model.

**Theorem 4.** *The equilibrium outcome in a market with multiple ad-based information platforms is the same as the one under full information disclosure given by*

$$x_i^{s*}(\theta) = \left[\frac{1}{2} + \frac{\Delta_\theta}{2}\right]_{[0,1]},$$

*i.e., at the equilibrium all platforms disclose perfect information about $\theta$.*

The result of Theorem 4 states that the competition among information platforms drives both platforms towards revealing their information about $\theta$ perfectly. We note that such

[4] We note that this is not the case in general. For instance, consider the example in Section V-A.

an outcome supports current outcomes observed in practice. That is, all major information platforms, such as Google Maps, Waze, or Apple Maps claim that they provide the best routing suggestion (*i.e.*, full information policy) to their users given their traffic information. We note that the result does not change if we consider a possibility that drivers can use both platforms since they do not acquire any additional information at the equilibrium.

It is known that perfect information disclosure may actually hurt social welfare [8], [9]. Therefore, compared to the scenario with a single platform, the competition among platforms may result in social welfare loss by disclosing full information to drivers and moving away from the partial information closure policy arising in a market with single platform. This is different from the analogous result in a market for a classical good, where competition among producers always results in higher social welfare. We demonstrate such a phenomenon through a numerical example in Section VII.

### B. Fee-based platforms

Next, we consider a duopoly market where both platforms charge a subscription fee to their users. Let $p_1$ and $p_2$ denote the subscription fee for platforms 1 and 2. In this case, the market equilibrium constraints are given by:

$$\begin{aligned} &U_0 \geq \max\{U_1 - p_1, U_2 - p_2\}, && \text{if } m_1 + m_2 < 1, \\ &U_1 \geq \max\{U_0, U_2 - p_2\}, && \text{if } m_1 > 0, \\ &U_2 \geq \max\{U_0, U_1 - p_1\}, && \text{if } m_2 > 0. \end{aligned}$$

The following theorem summarizes the result for a competitive market with multiple fee-based platforms.

**Theorem 5.** *Consider a market with multiple information platforms generating revenue by collecting subscription fee. There exists a unique equilibrium where*

*i) all platforms charge zero for their subscription fee, i.e.,* $p_1 = p_2 = 0$ *and*

*ii) all platforms disclose full information about* $\theta$*, i.e.,*

$$x_i^{s*}(\theta) = \left[\frac{1}{2} + \frac{\Delta_\theta}{2}\right]_{[0,1]}.$$

The result of Theorem 5 is similar to that of Theorem 4. That is, even when platforms charge their users for access to information, the competition among them forces them to reveal their information perfectly, and provide that information at no cost *i.e.*, $p_1 = p_2 = 0$. Following an argument similar to the one given in Section VI-B, such a competition among fee-based platforms can result in social welfare loss as it forces the platforms to reveal their information perfectly; see Section VII.

### C. Ad-based and fee-based platforms

The results of Theorems 4 and 5 can be extended to a market with both ad-based and fee-based platforms. The proof follows from the result of Theorems 4 and 5.

**Proposition 1.** *In a market with multiple information platforms (ad-based and/or fee-based), there exists a unique equilibrium outcome where (i) all platforms disclose their information perfectly, and (ii) all fee-based platforms with some users charge zero for their subscription fees.*

We would like to point out that the result of Proposition 1 implies that a fee-based platform cannot make a profit when it faces competition from another platform. This is not the case for an ad-based platform since it generates its revenue from advertisement and information gathering. Moreover, even though a social planner can improve the congestion by disclosing partial information rather full information to drivers [14]–[18], for-profit information platforms prefer to disclose their information perfectly at the equilibrium when they face competition. This is consistent with the market equilibrium observed in practice where all major navigation applications such as Google Maps, Waze, or Apple Maps do not charge any fee and provide their best routing suggestion to their users. We note that, in practice, the share of fee-based navigation services is not significant. Most of them are offered either (i) as an upgrade option for in-dash navigation systems (such as TomTom) that exploits the complementarity comfort between their free and fee-based services, or (ii) a specialized service targeting a very specific demographic and market segment that does face much competition.

Our result is also applicable to scenarios with a single dominant platform where the cost of entry for a new platform is not significant. In this case, the dominant platform finds it optimal to choose an ad-based business model and provides full information to its user to deter a potential competitor from entering the market. We would like to acknowldege that our model we do not capture drivers' privacy concern that a platform collects their information or their inconvenience with unwanted advertisements. Moreover, we assume that the platforms market is competitive and the cost of entrance is not significant. The consideration of these issues can potentially impact our results.

## VII. EXAMPLE

We provide a numerical example and compare the market outcomes for different business models and number of platforms. Let $\theta \in \{1, 3.5\}$, each with probability 0.5, and $a = 2$.

**No information:** We have $\Delta = \mathbb{E}\{\theta\} - a = 0.25$. Therefore, the no information outcome given by (3)-(4) is

$$x^s = 0.625, \quad W = 1.625.$$

**Social planner:** We have $\sigma^2 = 1.25^2 \geq 2|\Delta| - \Delta^2 = \frac{7}{16}$. Therefore, by Theorem 1, the efficient routing outcome, given by (7), is implementable by a social planner where

$$x^s(1) = 0.75, \quad x^s(3.5) = 0.125, \quad W = 1.83.$$

**Single ad-based platform:** We determine the optimal recommendation policy by maximizing $U_1 - U_0$ and solve the optimization problem (16). The optimal policy and the resulting social welfare are given by,

$$x^s(1) = 0.6875, \quad x^s(3.5) = 0.0625, \quad W = 1.82.$$

**Single fee-based platform:** The optimal recommendation policy is determined by solving the optimization problem (20). Consistent with part (ii) of Theorem 3, the optimal policy only covers $m = 0.727\%$ of all drivers and we have

$$x^s(1) = 0.937, \quad x^s(3.5) = 0.0625, \quad W = 1.82.$$

**Multiple platforms (full information):** By Proposition 1, the recommendation policy at the equilibrium is a full information disclosure policy. The outcome, characterized by (5)-(6), is given by

$$x^s(1) = 1, \quad x^s(3.5) = 0, \quad W = 1.75.$$

The example above provides an instance where the competition between platforms results in a social welfare loss compared to the outcome in the scenario with a single platform. Moreover, it also demonstrates that when platforms are strategic and profit-seeking, the resulting social welfare is inferior to the one achievable by a social planner.

## VIII. Conclusion

We investigated the outcome of a routing game when drivers can receive information from for-profit information platforms about the condition of every route in the network. We considered two business models for a platform. Under the first model, each platform seeks to generate revenue from advertisement and user information collection; under the second model, each platform charges its users a subscription fee. We showed that there exist instances where the traffic outcome is worse when multiple platforms exist compared to that when only a single platform is present. This is in contrast to the standard results for classical goods in economics. Moreover, we showed that in a competitive environment, or when the cost of market entry is not significant, it is optimal for an information platform to reveal its information completely to all drivers for free. Consequently, in a competitive environment all major information platforms must have an ad-based business model as it is the only profitable business model. This result is consistent with the current practice in the real-world where all major navigation applications are ad-based and free.

## References


[1] eMarketer. Smartphone penetration rate as share of the population in the United States from 2010 to 2021*s. www.themanifest.com/app-development/popularity-google-maps-trends-navigation-apps-2018, July 10, 2018.

[2] Riley Panko. The popularity of google maps: Trends in navigation apps in 2018. www.statista.com/statistics/201183/forecast-of-smartphone-penetration-in-the-us/, May 29, 2018.

[3] MarketsandMarkets. In-dash navigation system market by technology, by components, by connected navigation services, by vehicle type and by region - global forecast to 2022. www.marketsandmarkets.com/PressReleases/in-dash-navigation-system.asp, July 17, 2017.

[4] Richard Arnott, Andre De Palma, and Robin Lindsey. Does providing information to drivers reduce traffic congestion? *Transportation Research Part A: General*, 25(5):309–318, 1991.

[5] André De Palma, Robin Lindsey, and Nathalie Picard. Risk aversion, the value of information, and traffic equilibrium. *Transportation Science*, 2012.

[6] Robin Lindsey, Terry Daniel, Eyran Gisches, and Amnon Rapoport. Pre-trip information and route-choice decisions with stochastic travel conditions: Theory. *Transportation Research Part B: Methodological*, 2014.

[7] Jérôme Thai, Nicolas Laurent-Brouty, and Alexandre M Bayen. Negative externalities of gps-enabled routing applications: A game theoretical approach. In *19th International Conference on Intelligent Transportation Systems (ITSC)*. IEEE, 2016.

[8] Daron Acemoglu, Ali Makhdoumi, Azarakhsh Malekian, and Asuman Ozdaglar. Informational braess paradox: The effect of information on traffic congestion. *Operations Research*, 66(4):893–917, 2018.

[9] Zaid Khan and Saurabh Amin. Bottleneck model with heterogeneous information. *Transportation Research Part B: Methodological*, 2018.

[10] S. Hendrix (The Washington Post). Traffic-weary homeowners and Waze are at war, again. guess who's winning? June 5, 2016.

[11] L. Nelson (Los Angeles Times). New traffic apps may be pushing cars into residential areas. www.latimes.com/local/california/la-me-california-commute-20150106-story.html, January 5, 2015.

[12] G. Richards (The Mercury News). Is Googles Waze app making traffic worse? www.mercurynews.com/2016/01/21/is-googles-waze-app-making-traffic-worse/, January 21, 2016.

[13] K. Brunhuber (NBC Miami). Shortcut-finding app Waze creating residential traffic headaches. www.cbc.ca/news/technology/shortcut-app-waze-1.3463004, February 29, 2016.

[14] Hamidreza Tavafoghi and Demosthenis Teneketzis. Informational incentives for congestion games. In *55th Annual Allerton Conference on Communication, Control, and Computing*, 2017.

[15] Sanmay Das, Emir Kamenica, and Renee Mirka. Reducing congestion through information design. In *55th Annual Allerton Conference on Communication, Control, and Computing*, 2017.

[16] Lichun Li, Olivier Massicot, and Cedric Langbort. Sequential public signaling in routing games with feedback information. In *IEEE Conference on Decision and Control (CDC)*, 2018.

[17] Olivier Massicot and Cedric Langbort. Public signals and persuasion for road network congestion games under vagaries. *IFAC-PapersOnLine*, 51(34):124–130, 2019.

[18] Y. Liu and A. B Whinston. Efficient real-time routing for autonomous vehicles through bayes correlated equilibrium: An information design framework. *Information Economics and Policy*, 2019.

[19] Emir Kamenica and Matthew Gentzkow. Bayesian persuasion. *The American Economic Review*, 101(6):2590–2615, 2011.

[20] Dirk Bergemann and Stephen Morris. Information design, bayesian persuasion, and bayes correlated equilibrium. *American Economic Review*, 106(5):586–91, 2016.

[21] Yan Leng, Larry Rudolph, Alex'Sandy' Pentland, Jinhua Zhao, and Haris N Koutsopolous. Managing travel demand: Location recommendation for system efficiency based on mobile phone data. *arXiv preprint arXiv:1610.06825*, 2016.

[22] Hamidreza Tavafoghi, Akhil Shetty, Kameshwar Poolla, and Pravin Varaiya. Strategic information platforms in transportation networks (extended version), 2019. www-personal.umich.edu/%7Etavaf/allerton19_extended.pdf.

[23] Hamidreza Tavafoghi. *On design and analysis of cyber-physical systems with strategic agents*. PhD thesis, Ph. D. dissertation, University of Michigan, Ann Arbor, 2017.

[24] Tim Roughgarden. Routing games. *Algorithmic game theory*, 18:459–484, 2007.

[25] Larry G Epstein and Michael Peters. A revelation principle for competing mechanisms. *Journal of Economic Theory*, 1999.

## APPENDIX

*Proof of Lemma 1.* The first part of the lemma directly follows from the definition of the obedience constraints. We have,

$$\sum p_i x(\theta_i) u^S(\theta_i) \geq \sum p_i x(\theta_i) u^R(\theta_i),$$
$$\sum p_i (1 - x(\theta_i)) u^R(\theta_i) \geq \sum p_i (1 - x(\theta_i)) u^S(\theta_i).$$

Therefore,

$$\begin{aligned} U_1 &= \sum p_i \left[x(\theta_i) u^S(\theta_i) + (1 - x(\theta_i)) u^R(\theta_i)\right] \\ &\geq \sum p_i \left[x(\theta_i) u^S(\theta_i) + (1 - x(\theta_i)) u^S(\theta_i)\right] = \mathbb{E}\{u^S(\theta)\} \\ &\geq \sum p_i \left[x(\theta_i) u^R(\theta_i) + (1 - x(\theta_i)) u^R(\theta_i)\right] = \mathbb{E}\{u^R(\theta)\}. \end{aligned}$$

Thus, $U_1 \geq \max\{\mathbb{E}\{u^S(\theta)\}, \mathbb{E}\{u^R(\theta)\}\} = U_0$, and all drivers weakly prefer to join the platform.

Next we prove the second part of the Lemma.

(Only if part): We first prove that if both obedience constraints are satisfied as strict inequality, then $U_0 < U_1$. That is, at equilibrium, each driver strictly prefer to join. For obedience constraints, we have:

$$\sum p_i x(\theta_i) u^S(\theta_i) > \sum p_i x(\theta_i) u^R(\theta_i),$$
$$\sum p_i (1 - x(\theta_i)) u^R(\theta_i) > \sum p_i (1 - x(\theta_i)) u^S(\theta_i),$$

for obedience constraints for recommendations to take $S$ and $R$, respectively.

Therefore,

$$\begin{aligned} U_1 &= \sum p_i \left[x(\theta_i) u^S(\theta_i) + (1 - x(\theta_i)) u^R(\theta_i)\right] \\ &> \sum p_i \left[x(\theta_i) u^S(\theta_i) + (1 - x(\theta_i)) u^S(\theta_i)\right] = \mathbb{E}\{u^S(\theta)\} \\ &> \sum p_i \left[x(\theta_i) u^R(\theta_i) + (1 - x(\theta_i)) u^R(\theta_i)\right] = \mathbb{E}\{u^R(\theta)\}. \end{aligned}$$

Thus, $U_1 > \max\{\mathbb{E}\{u^S(\theta)\}, \mathbb{E}\{u^R(\theta)\}\} = U_0$.

(If part): Assume that each driver strictly prefer to join, but there exists an obedience constraints that is satisfied as an equality; without loss of generality, assume that this the first obedience constraint for route $S$. That is,

$$\sum p_i x(\theta_i) u^S(\theta_i) = \sum p_i x(\theta_i) u^R(\theta_i),$$
$$\sum p_i (1 - x(\theta_i)) u^R(\theta_i) \geq \sum p_i (1 - x(\theta_i)) u^S(\theta_i).$$

Then, we have,

$$\begin{aligned} U_1 &= \sum p_i \left[x(\theta_i) u^S(\theta_i) + (1 - x(\theta_i)) u^R(\theta_i)\right] \\ &= \sum p_i \left[x(\theta_i) u^R(\theta_i) + (1 - x(\theta_i)) u^R(\theta_i)\right] = \mathbb{E}\{u^R(\theta)\}, \\ &> \sum p_i \left[x(\theta_i) u^S(\theta_i) + (1 - x(\theta_i)) u^S(\theta_i)\right] = \mathbb{E}\{u^S(\theta)\} \end{aligned}$$

Therefore, $U_0 = \max\{\mathbb{E}\{u^S(\theta)\}, \mathbb{E}\{u^R(\theta)\}\} = \mathbb{E}\{u^R(\theta)\} = U_1$, which is a contradiction. □

*Proof of Theorem 2.* The proof of Theorem 2 follows directly from the proof of part (i) of Theorem 5. Set $m = 1$. Then the optimization problem 16 is identical to the optimization problem 20 when $m = 1$. From part (i) of Theorem 5, it is optimal for the fee-based platform to implement the socially optimal recommendation policy. □

*Proof of Theorem 3.* **Part(i):** By the symmetry assumption, the distribution of $\theta$ is symmetric around $a$. That is, $\theta_i - a = -(\theta_{N+1-i} - a)$ and $p_i = p_{N+1-i}$ for $1 \leq i \leq N$. Without loss of generality assume that $N$ is an odd number (otherwise add a realization $\theta = a$ with probability 0). Consider the following change of variable from $\theta$ to $\tilde{\theta}$ such that $\tilde{\theta}_j := \theta_{\frac{N+1}{2}+j} - a$ and $\tilde{\theta}_{-j} = \theta_{\frac{N+1}{2}-j} - a$ for $0 \leq j \leq \frac{N-1}{2}$. We have $\tilde{\theta}_j = \tilde{\theta}_{-j}$ and $\mathbb{P}\{\theta = \tilde{\theta}_j + a\} = \mathbb{P}\{\theta = \tilde{\theta}_{-j} + a\}$.

Similarly, consider the change of variable from $x(\theta)$ to $\tilde{x}(\tilde{\theta})$ as $\tilde{x}(\tilde{\theta}_j) := \frac{1}{2} - x(\theta_{\frac{N+1}{2}+j})$ and $\tilde{x}(\tilde{\theta}_{-j}) := \frac{1}{2} - x(\theta_{\frac{N+1}{2}-j})$ for $0 \leq j \leq \frac{N-1}{2}$. We can rewrite the optimization problem (20) as

$$\max_{p,m,\tilde{x}(\tilde{\theta})} \quad pm$$

subject to

$$\begin{aligned} & m \in [0,1], \tilde{x}(\tilde{\theta}) \in [-\frac{1}{2}, -\frac{1}{2}], \\ & u^S(\tilde{\theta}_i) = a + \left[-\left(\frac{1}{2} - \tilde{x}(\tilde{\theta}_i)\right) m - y(1-m)\right] \\ & u^R(\tilde{\theta}_i) = a + \left[\tilde{\theta}_i - \left(\frac{1}{2} + \tilde{x}(\tilde{\theta}_i)\right) m - (1-y)(1-m)\right] \\ & U^S = \sum_{-\frac{N+1}{2}}^{\frac{N+1}{2}} p(\tilde{\theta}_i) u^S(\tilde{\theta}_i), \\ & U^R = \sum_{-\frac{N+1}{2}}^{\frac{N+1}{2}} p(\tilde{\theta}_i) u^R(\tilde{\theta}_i), \\ & U_0 \geq U^S, \\ & U_0 \geq U^R, \\ & \sum p(\tilde{\theta}_i) \left(\frac{1}{2} - \tilde{x}(\tilde{\theta}_i)\right) \left[u^S(\tilde{\theta}_i) - u^R(\tilde{\theta}_i)\right] \geq 0, \quad \text{(obed. for } S\text{)} \\ & \sum p(\tilde{\theta}_i) \left(\frac{1}{2} + \tilde{x}(\tilde{\theta}_i)\right) \left[u^R(\tilde{\theta}_i) - u^S(\tilde{\theta}_i)\right] \geq 0. \quad \text{(obed. for } R\text{)} \\ & U_1 = \sum_{-\frac{N+1}{2}}^{\frac{N+1}{2}} p(\tilde{\theta}_i) \left[\left(\frac{1}{2} - \tilde{x}(\tilde{\theta}_i)\right) u^S(\tilde{\theta}_i) + \left(\frac{1}{2} + \tilde{x}(\tilde{\theta}_i)\right) u^R(\tilde{\theta}_i)\right], \\ & p = U_1 - U_0, \end{aligned}$$

where we write $U_0 = \max\{U^S, U^R\}$ as $U_0 \geq U^S$ and $U_0 \geq U^R$. In the following, we first show that at the optimal solution we must have $\tilde{x}^*(\tilde{\theta}_j) = -\tilde{x}^*(\tilde{\theta}_{-j})$ and $y^* = \frac{1}{2}$.

Let $m^*$, $y^*$, and $\left(x^*_{-\frac{N+1}{2}}, \ldots, x^*_{\frac{N+1}{2}}\right)$ denote an optimal solution. Because of the symmetry between $\tilde{\theta}_i$ and $\tilde{\theta}_{-i}$ in the optimization problem between , it is easy to verify that if

$$\left(\tilde{x}(\tilde{\theta}_{-\frac{N+1}{2}}),\ldots,\tilde{x}(\tilde{\theta}_{\frac{N+1}{2}})\right)=\left(x^*_{-\frac{N+1}{2}},\ldots,x^*_{\frac{N+1}{2}}\right)$$

is an optimal solution, then

$$\left(\tilde{x}(\tilde{\theta}_{-\frac{N+1}{2}}),\ldots,\tilde{x}(\tilde{\theta}_{\frac{N+1}{2}})\right)=\left(-x^*_{\frac{N+1}{2}},\ldots,-x^*_{-\frac{N+1}{2}}\right)$$

is also an optimal solution along with $m^*$, $-y^*$.

Consider an average of the above solutions given by $m^*$, $\frac{1}{2}$, and

$$\left(\tilde{x}(\tilde{\theta}_{-\frac{N+1}{2}}),\ldots,\tilde{x}(\tilde{\theta}_{\frac{N+1}{2}})\right) = \left(\frac{x^*_{-\frac{N+1}{2}}-x^*_{\frac{N+1}{2}}}{2},\ldots,\frac{x^*_{-\frac{N+1}{2}}-x^*_{\frac{N+1}{2}}}{2}\right).$$

It is clear that the average solution does not affect the linear constraints (the first seven constraints). Moreover, it also satisfies the obedience constraints. This is because we can write the obedience constraints for the average solution by considering duplicate realizations for each $\tilde{\theta}^j_i$, $j=1,2$ where each one has $p(\tilde{\theta}^j_i)=\frac{1}{2}p(\tilde{\theta}_i)$; for $j=1$ one we make recommendation based on the first solution and for $j=2$ we make recommendations based on the second one. Therefore, the obedience constraints for the average solution is simply the average of the obedience constraints for the original solutions. Thus the obedience constraints are also satisfied for the average solution.

Now, consider the equation for $U_1$, which is a quadratic function in $\tilde{x}(\tilde{\theta}_i)$. Consider the quadratic terms including $\tilde{x}(\tilde{\theta}_i)$ and $\tilde{x}(\tilde{\theta}_{-i})$. Using CauchySchwarz inequality, it is easy to verify that if $\tilde{x}^*(\tilde{\theta}_i)\neq-\tilde{x}(\tilde{\theta}_{-i})$ then the average solution constructed above results in higher $U_1$, and thus, $p$ is higher since $p=U_1-U_0$. Therefore, at an optimal solution we must have $y^*=\frac{1}{2}$ and $\tilde{x}^*(\tilde{\theta}_i)=-\tilde{x}(\tilde{\theta}_{-i})$.

Given that $\tilde{x}^*(\tilde{\theta}_i)=-\tilde{x}(\tilde{\theta}_{-i})$, it is easy to show that $U_0=\frac{a+\mathbb{E}\{\theta\}-1}{2}$ for every $m$. Therefore, the platform objective is to maximize $m\left(U_1-U_0\right)$. Therefore, given $m$, the platform maximizes $U_1$, *i.e.* the welfare of its users. Thus, given $m$, we have

$$\tilde{x}^*(\tilde{\theta}_i)=\left[\frac{\tilde{\theta}_i}{4m}\right]_{[-\frac{1}{2},\frac{1}{2}]}$$

**Part(ii):** Writing $U_1$ explicitly in terms of $m$ and $\tilde{x}^*(\tilde{\theta}_i)$, using $y=\frac{1}{2}$, we have

$$U_1=a-\frac{1}{2}+\sum_{-\frac{N+1}{2}}^{\frac{N+1}{2}}\tilde{\theta}_i\left[\frac{\tilde{\theta}_i}{4m}\right]_{[-\frac{1}{2},\frac{1}{2}]}-2m\left[\frac{\tilde{\theta}_i}{4m}\right]^2_{[-\frac{1}{2},\frac{1}{2}]}$$

Therefore, we can rewrite the objective function as

$$mp(m)=U_1-U_0=m\sum_{-\frac{N+1}{2}}^{\frac{N+1}{2}}\tilde{\theta}_i\left[\frac{\tilde{\theta}_i}{4m}\right]_{[-\frac{1}{2},\frac{1}{2}]}-2m\left[\frac{\tilde{\theta}_i}{4m}\right]^2_{[-\frac{1}{2},\frac{1}{2}]}.$$

The result of part (ii) follows from the first order condition. □

***Proof of Theorem 4.*** Suppose the equilibrium outcome is different from the full information disclosure outcome. Then, $|u^s(\theta)-u^r(\theta)|$ is non-zero for some $\theta\in\Theta$. This implies that there are users who could increase their expected utility if they had access to a recommendation policy equivalent to revealing more information. That is, a platform can attract them by recommending the better route for those $\theta$. Therefore, this is not an equilibrium outcome as at least one platform can do better by revealing more information to such users. □

***Proof of Theorem 5.*** **Step 1:** We first prove that $p_1=p_2=0$ at an equilibrium.

Suppose this is not the case. Then, $p_i>0$ at an equilibrium for some $i$.

Next, given that $p_i>0$, we show that $m_j>0$ and $p_j>0$ for some $j$ at equilibrium.

If $m_i=0$, there two cases: (i) $m_j=0$, and (ii)$m_j>0$. In case (i), platform $j$ can disclose the perfect the information about $\theta$ and set $p_j=p_i-\epsilon$ for small enough $\epsilon$. Then, all drivers strictly prefer to join platform $j$, and platform $j$ can make a positive profit by such a deviation. Therefore, case (i) is not possible in an equilibrium. In case (ii), consider the following scenarios where: (ii-1) $p_j=0$ and (ii-2) $p_j>0$.

For case (ii-1), platform $j$ can disclose the perfect the information about $\theta$ and set $p_j=p_j-\epsilon$ for small enough $\epsilon$. Thus, platform $j$ can make a positive profit by such a deviation. Therefore, case (ii-1) is not possible in an equilibrium. Under case (ii-2), we have $m_j>0$ and $p_j>0$, and thus, the above condition is satisfied.

Without loss of generality assume that $j=1$. Then, there are two cases: (a) $m_2>0$ and (b) $m_2=0$.

Consider case (a) where $m_2>0$. Then, we have $U_1-p_1=U_2-p_2$. Consider strategy $(\hat{x}^s_1,\hat{p}_1)$ for platform 1 such that $\hat{p}_1:=p_1-\epsilon$ for small enough $\epsilon$, and

$$\hat{x}^s_1:=\begin{cases}\tilde{x}^s_1, & w.p.\ \frac{m_1}{m_1+m_2}\\ \tilde{x}^s_2, & w.p.\ \frac{m_2}{m_1+m_2}\end{cases}$$

Then, we have $\tilde{U}_1-\tilde{p}_1>U_2-p_2$. Thus, all of the users served by platform 2 will do better by switching to platform 1. Thus, case (a) cannot arise in an equilibrium.

Consider case (b). Then platform 2, can disclose the same information as platform 1 and set $p_2=p_1-\epsilon$ for small enough $\epsilon$. Then, all users of platform 1 strictly prefer to join platform 2. Therefore, case (b) also cannot arise in an equilibrium. Therefore we have $p_1=p_2=0$ at an equilibrium.

**Step 2:** We show that at an equilibrium the outcome is the same as the full information disclosure outcome. Assume that this is not the case. Then, there exists $\theta_i$ such that $u^s(\theta_i)\neq u^r(\theta_i)$. Then platform 1 can make a positive profit by disclosing $\theta_i$ perfectly to small enough population size, at price $p_1=\epsilon$ for small enough $\epsilon$. Therefore, at an equilibrium, the outcome must be identical to the full information disclosure outcome. □

***Proof of Proposition 1***. We present the proof for the case with one ad-base platform (platform 1) and one fee-based platform (platform 2). The result for a more general case follows from the result of Theorem Theorems 2 and 3.

Let $m_2 > 0$. Then, we must have platform 1 disclose perfect information about $\theta$ and $p_2 = 0$. Otherwise, platform 2 can disclose perfect information about $\theta$ and attract all drivers since it provides that information for free.

Now, we show that if $m_2 = 0$, then platform 1 must disclose perfect information. Assume that this is not the case. Then, there exists $\theta_i$ such that $u^s(\theta_i) \neq u^r(\theta_i)$. Then platform 2 can make a positive profit by disclosing $\theta_i$ perfectly to small enough population size, at price $p_1 = \epsilon$ for small enough $\epsilon$.

Therefore, at an equilibrium we must have $p_2 = 0$ if $m_2 > 0$. Moreover, the outcome is the same as the full information disclosure outcome.

$\square$