Strategic Information Provision in Routing Games

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Abstract

We investigate the problems of designing public and private information disclosure mechanisms by a principal in a transportation network so as to improve the overall congestion. We show that perfect disclosure of information about the routes' conditions is not optimal, and paradoxically, may in fact increase congestion. The principal can improve the congestion (*i.e.* social welfare) by providing either a public imperfect information signal about road conditions, or alternatively, coordinated routing recommendation to drivers based on the road conditions. When the uncertainty about the road conditions is high relative to the ex-ante difference in the road conditions (*i.e.,*the value of information is high), we show that the socially efficient routing outcome is achievable using a private information disclosure mechanism. Furthermore, we study the problem of optimal dynamic private information disclosure mechanism design in a dynamic two-time step setting. We consider different pieces of information that drivers may observe and learn from at $t = 1$, and investigate qualitative properties of an optimal dynamic information disclosure mechanisms.

I. Introduction

A. Background and Motivation

In a transportation network, the condition of every link varies over time due to changes in weather conditions, accidents, traffic jams, etc. Traditionally, drivers receive public information about these changes at every route through various infrastructures, *e.g.* regional traffic updates via radio broadcasts, and/or variable (dynamic) message signs on road sides displaying specific information about the onward routes [2]–[4]. In recent years, the advent of GPS-enabled routing

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devices and navigation applications (*e.g.* Waze and Google maps) has enabled drivers to receive private, real-time data about the transportation network's condition for their own intended origin-destination [5]. The development of these technologies creates new opportunities to reduce congestion in the network, and improve its overall performance (as measured by various metrics including *social welfare*).

Several studies have investigated the effects of information provision to drivers on the social welfare of the transportation network [6]–[13]. These studies have shown that the effect of information provision on social welfare is ambiguous, and in general, is not necessarily socially beneficial. For exogenously fixed information provision structures, these works have identified instances where the provision of information can in fact increase congestion in parts of the network, leading to a decrease in social welfare. In addition to the above-mentioned theoretical and experimental works, there are empirical evidences that identify negative impacts of information provision on the network's congestion [14]–[18]. Therefore, it is important to investigate how to design appropriate information provision mechanisms, in a manner that is socially beneficial and leads to a reduction in the overall congestion in the transportation network.

In this paper, we study the problem of designing a socially optimal information disclosure mechanism. We consider a congestion game [19], [20] in a parallel two-link network. We consider an information provider (principal) who wants to disclose information about the condition of the network to a fixed population of drivers (agents). We assume that the condition of one route/link, called safe route, is constant and known to everyone, while the condition of the other route, called risky route, is random and only known to the principal. The principal wants to design an information disclosure mechanism so as to maximize the social welfare.

We study the problem of designing an optimal information provision mechanism in two cases: (1) when the principal can only provide information that is publicly available to all drivers, and (2) when the principal can provide private information to each driver individually.

We first consider a static setting where the drivers do not learn from their past experiences. We determine a condition under which the principal can achieve the maximum social welfare using an optimal information provision mechanism. That is, the principal can utilize her *superior information* about the network to provide *informational incentives* so as to align the drivers' objectives with the overall social welfare. Next, we consider a dynamic two-stage setting where the drivers learn from their experience at $t = 1$, and the risky routes' condition evolves according to an uncontrolled Markov chain. We consider three scenarios for the drivers' learning at $t = 1$:

(i) the drivers only learn from the information they receive directly from the principal at $t = 1$, (ii) in addition to the information they receive directly from the principal at $t = 1$, the drivers who take the risky route learn perfectly its condition at $t = 1$, and (iii) in addition to the information he receives directly from the principal at $t = 1$, each driver perfectly observes the number of cars/drivers on the route he takes at $t = 1$. Using numerical simulations, we show that in scenario (i) the principal can achieve the same outcome as in the static setting in which there is no learning. However, in scenarios (ii) and (iii) the performance of an optimal dynamic information provision mechanism decreases due to the drivers' learning only. In particular, we identify instances in scenario (ii) where it is optimal for principal to reveal perfectly risky route's condition at $t = 2$ so that the drivers do not have an incentive to experiment and learn the risky route's condition at $t = 1$. Moreover, in scenario (iii) we identify instances where it is optimal for the designer to not implement different routing outcome, *i.e.* reveal her information about risky route's condition, so as to increase her information superiority at $t = 2$.

B. Related literature

The problems investigated in [6]–[13], [21] are the most closely related to our problem. The authors of [6], [7] consider a *bottleneck* model [22] with stochastic capacities, where each route is modeled as a queue with a first-come, first-served policy, with the service rate determined by the route's condition. In [6], the authors consider a scenario where each driver decides the time and the route/queue he wants to join, considering the behavior of the other drivers. Through numerical simulations, they show that when drivers receive a low quality/highly noisy signal about the routes' conditions, social welfare decreases. In [7], the authors consider a scenario where each driver does not take into account the other drivers' response to the information provided by the principal. They show that social welfare may decrease when the drivers receive accurate information about the routes, due to the drivers' *overreaction* and/or higher *congestion concentration* in parts of the network.

The authors of [8], [9] consider a parallel two-link network that is similar to our model. In [8], the authors assume that only one of the routes has a random condition with two possible values. They show that social welfare may decrease when the drivers receive public information about the routes' conditions irrespective of whether they are risk-neutral or risk-averse. The authors of [9] assume that the condition of both routes are random. They show that social welfare can decrease when the drivers receive public perfect information about the realizations of the routes' conditions.

The authors of [10]–[12] consider a model where a subset of drivers (informed drivers) has access to more accurate information about the condition of every route than the remaining drivers. In [10], the authors assume that the drivers that do not receive the more accurate information prefer to use high-capacity routes (*i.e.* highways) rather than low-capacity routes. By numerical simulation, they show that as the number of informed drivers increases, the congestion in low-capacity routes (*i.e.* urban areas) increases. The authors of [12] consider a model similar to ours, in which the condition of one of the routes is random. They show that when the number of informed drivers is low, the expected utilities of both groups of drivers are higher compared to the case where all drivers are uninformed. However, as the number of informed drivers increases, the social welfare decreases, even compared to the case where all drivers are uninformed. The authors of [11] study a model where the informed drivers become aware of the existence of additional routes in a network. They show that the provision of information can create a *Braess' Paradox* phenomenon, and thus, can reduce not only the social welfare, but also the utility of the informed drivers.

In contrast to [6]–[13], [21], which analyze the performance of *fixed* information provision mechanisms, in this paper we investigate the problem of designing an optimal information provision mechanism; the information provision mechanisms analyzed in [6]–[13], [21] are within the set of feasible mechanisms the principal can choose from when maximizing the social welfare.

Our work is also related to the literature on improving efficiency in resource allocation problems with externalities (*i.e.* congestion games). It is known that the equilibrium outcome in congestion games is not socially optimal [23]. Several approaches have been proposed in the literature to address this inefficiency. One approach is to utilize monetary mechanisms in order to align the agents' objectives with the social welfare (see [24] and references therein). A second approach, which is applicable when the principal has control over a fraction of agents, is for the principal to choose routes for this fraction so as to influence the behavior of selfish agents. This can lead to improvement in the social welfare (see [25], [26] and references therein). We propose an alternative approach to improving the efficiency in congestion game by utilizing informational incentives when the principal has an informational advantage over the agents. Specifically, our approach can prove promising in transportation networks where the application of tolls (pricing) is limited and agents are typically selfish.

Within the economics literature, the problems studied in this paper belong to the class of information design problems (see [27] and references therein). Our approach to the public information mechanism design problem (Section IV) is similar to the ones in [28], [29]. Our approach to the private information mechanism design problems (Section V and VI) is similar to the ones in [30], [31]. The work in [32] is closely related to our work. The authors of [32] consider a model with two possible actions where the payoff of one of the actions is not known, even to the principal. The principal (*e.g.* the Waze application) faces a group of short-lived agents that arrive sequentially over time. She wants to design an information mechanism that provides information about the agents' past experience to the incoming agents over time. Our work is different from [32] as (i) in contrast to the single-agent decision problem considered in [32], our model assumes that the principal faces a population of agents that create negative externalities on one other at each time step, and (ii) in the dynamic setting, agents are long-lived and learn from their past (private) experience, while in [32], the agents are short-lived.

The dynamic two-stage problems studied in this paper are also related to literature on strategic experimentation in economics [33]–[35]. The authors of [36] study a monetary mechanism design in a principal-agent relationship. Our problem is different from that of [36] since we study an information disclosure mechanism design instead of a monetary mechanism design. The authors of [37], [38] study the problem of information disclosure mechanism design in an innovation contest. Our problem is different from those of [37], [38] since in contrast to their models where agents do not observe each others' actions and payoffs unless the principal discloses information and an agent's payoff is independent of other agents' action, in our model agents' actions create negative externality and influence agents' payoffs, and each agent may have an indirect observation about other agents' actions and payoffs at $t = 1$.

The information design problems that we consider in this paper are also related to the problem of designing real-time communication systems [39]–[42]. However, in contrast to these studies, where the receivers are cooperative and have the same objective as the transmitter, in our problem the drivers are strategic and have objectives that are different from that of the principal. The authors of [43] consider a problem of real-time communication with a strategic transmitter and receiver, Gaussian source, and quadratic estimation cost. They follow an approach that is similar to that of [29] and our approach for the design of public information disclosure mechanism in Section IV. However, our problem is different from that of [43] since in our model there exist many agents, and each agent's utility depends on his action and the routes' conditions as well as other agents' actions. Moreover, in this paper, we study the problem of private information disclosure mechanism design that is not present in the work of [43].

C. Contribution

We determine optimal public and private information provision mechanisms that maximize the social welfare in a transportation network. Our results propose a solution to the concern raised in [6]–[13], [21] about the potential negative impact of information provision on congestion in transportation networks. We show that the principal can utilize his superior information about the condition of the network, and provide informational incentives to the drivers so as to improve the social welfare. When the principal can disclose information to every driver privately, we show that the principal can benefit from providing coordinated routing recommendations to the drivers. We identify a condition under which the principal can achieve the efficient routing outcome in a static setting. Moreover, we consider a dynamic setting with two-time steps under three scenarios, each capturing a possible piece of information that the drivers can learn from it in dynamic setting. Using numerical simulations, we discuss the effect of each piece of information on the solution to the optimal dynamic information mechanism and its qualitative properties.

D. Organization

The rest of this paper is organized as follows. In Section II, we present our model in a static setting. In Section III, we consider two naive information mechanisms and compare their outcomes with the socially efficient outcome. We study the problem of designing an optimal public information mechanism in Section IV. In Section V, we study the problem of designing an optimal private information mechanism. We consider the design of optimal dynamic information mechanisms in a two-step setting in Section VI, and investigate the effect of different types of drivers' observations on the performance and qualitative properties of an optimal dynamic mechanism through numerical simulations. We conclude in Section VII. All proofs appear in the Appendix.

II. Model

Consider a two-link network managed by a principal who wants to maximize social welfare (Figure 1). There is a unit mass of agents traveling from the origin O to the destination D.

There are two routes/links that agents can take. The top route, denoted route s (*i.e.* safe route) has condition $a > 0$ that is known to all agents and the principal. The bottom route, denoted route r (*i.e.* risky route), has a condition $\theta \in \Theta := \{\theta^1, \ldots, \theta^M\}$, $\theta^1 < \theta^2 < \ldots < \theta^M$, that is not known to the agents and is only known to the principal. It is common knowledge among the agents and the principal that θ takes values in $\{\theta^1, \ldots, \theta^M\}$ with probability $\{p_{\theta^1}, p_{\theta^2}, ..., p_{\theta^M}\}$, respectively. Let x^s and x^r (where $x^s + x^r = 1$), denote the mass of agents that choose route s and r, respectively. The utility of each agent depends on the condition of the route that he chooses to travel as well as on the congestion (negative externality) that he observes along his route. Given x^s and x^r, let $C^s(x^s)$ and $C^r(x^r)$ denote the congestion cost at route s and route r, respectively. The functions $C^s(\cdot)$ and $C^r(\cdot)$ are strictly increasing, with $C^s(0) = 0$ and $C^r(0) = 0$. For the ease of exposition, we assume that $C^s(x^s) = x^s$ and $C^r(x^r) = x^r$. Throughout this paper, we discuss how our results extend to general congestion functions.

We assume that the utility of an agent taking route s (resp. r) is given by $a - C^s(x^s) = a - x^s$ (resp. $\theta - C^r(x^r) = \theta - x^r$); that is, the effect of a route's condition on an agent's utility is separable from the effect of the congestion cost. Therefore, the expected social welfare W is given by

$$W := \mathbb{E}\left\{x^s(a - C^s(x^s)) + x^r(\theta - C^r(x^r))\right\}. \tag{1}$$

We make the following assumption about the possible values of θ.

Assumption 1. *The risky route's types θ are such that $\theta^M - C^r(1) \leq a$ and $a - C^s(1) \leq \theta^1$.*

Assumption 1 ensures that for every realization θ of route r's condition, there will be a positive mass of agents taking either route.

The principal wants to design an information disclosure mechanism that provides information about the condition θ of route r to the agents, so as to maximize the expected social welfare W. We consider two classes of information disclosure mechanisms by the principal: (i) public information disclosure mechanisms, where the principal sends a public signal about θ which is observed by all agents (Section IV), (ii) private information disclosure mechanisms, where the principal sends a private signal about θ to each agent, and this signal is only observed by that agent (Section V).

Before proceeding with the study of optimal public and private information disclosure mecha-

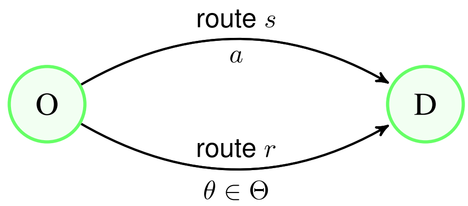


Fig. 1: The two-link network

nisms which maximize social welfare, we present two naive information disclosure mechanisms in Section III. By exploring the agents' routing decisions under these two naive information disclosure mechanisms, along with the socially efficient routing decisions, we will elaborate on the main insights underlying some of the results appearing in the rest of the paper.

III. Naive Mechanisms

We study two naive information disclosure mechanisms that the principal can employ to disclose information about the condition θ of route r, namely, the no information disclosure and full information disclosure mechanisms. We then present the socially optimal outcome and compare it to the outcomes of the naive mechanisms. Let $\mu := \mathbb{E}\{\theta\} = \sum_{\theta\in\Theta} p_\theta \theta$ denote the expected condition of route r. Define $\Delta := a - \mu$ and $\Delta_\theta := a - \theta$ as the expected and realized difference between the conditions of routes s and r. Let $\sigma^2 = \mathbb{E}\left\{(\theta-\mu)^2\right\}$ denote the variance of route r's condition. In the sequel, we characterize the traffic outcome under different information that the agents may receive as a function of μ, Δ, and Δ_θ.

A. No Information Disclosure

Consider an information disclosure mechanism where the principal discloses no information about θ to the agents. In this case, the expected utility from route r, given by $\mu - x^r$, must be equal to the utility $a - x^s$ from route s; this is because otherwise, some agents would switch from the route with lower utility to the one with higher utility[2]. Therefore, the traffic at routes

[2]Note that by Assumption 1, both routes are non-empty for any realization of the condition of the risky route.

s and r are given by

$$x^{s,\text{no info}} = \frac{1}{2} + \frac{1}{2}\Delta, \tag{2}$$

$$x^{r,\text{no info}} = \frac{1}{2} - \frac{1}{2}\Delta. \tag{3}$$

That is, the difference between the traffic on routes s and r depends on the expected difference between the routes' conditions, given by Δ.

Consequently, the expected social welfare $W^{\text{no info}}$ under the no information disclosure mechanism is given by

$$W^{\text{no info}} = \frac{a + \mu - 1}{2}, \tag{4}$$

where $\frac{a+\mu-1}{2}$ denotes the expected utility of an agent taking either of the routes.

B. Full Information Disclosure

Consider an information disclosure mechanism where the principal reveals perfectly the condition θ of route r to all agents. In this case, agents choose their route knowing θ. By an argument similar to the one given above for the no information disclosure mechanism, the utility from taking either of the routes must be equal. Therefore, the traffic at routes s and r are given by

$$x^{s,\text{full info}}(\theta) = \frac{1}{2} + \frac{1}{2}\Delta_\theta, \tag{5}$$

$$x^{r,\text{full info}}(\theta) = \frac{1}{2} - \frac{1}{2}\Delta_\theta. \tag{6}$$

In this case, the traffic difference between routes s and r depends on the *realized* difference Δ_θ between the routes' conditions, as opposed to the *expected* difference Δ that determines the outcome under the no information disclosure mechanism.

Using (5) and (6), we can obtain the expected social welfare $W^{\text{full info}}$ under the full information disclosure mechanism as

$$W^{\text{full info}} = \mathbb{E}\{\frac{a + \theta - 2}{2}\} = \frac{a + \mu - 1}{2}. \tag{7}$$

Remark 1. *We note that the expected social welfare $W^{\text{full info}}$ under the full information disclosure mechanism and $W^{\text{no info}}$ under the no information disclosure mechanism are the same in the model of Section II with linear congestion costs. This is because under the full information* *disclosure mechanism the social welfare is linear in θ. As we discuss in Remark 2 below, for congestion functions $C^s(x^s)$ and $C^r(x^r)$ that are nonlinear in x^s and x^r, respectively, the social welfares under the full information and no information disclosure mechanisms are not identical in general.*

C. Socially Efficient Outcome

When each agent chooses his route, under either the no information or full information disclosure mechanisms, he does not take into account the congestion (*i.e.* negative externality) that his decision creates on the other agents. Therefore, the social welfare under the no information and full information disclosure mechanisms are different from the one under the socially efficient outcome. The socially efficient routing outcome is given by

$$x^{s,\text{eff}}(\theta) = \frac{1}{2} + \frac{1}{4}\Delta_\theta, \tag{8}$$

$$x^{r,\text{eff}}(\theta) = \frac{1}{2} - \frac{1}{4}\Delta_\theta, \tag{9}$$

and the corresponding expected social welfare is given by

$$W^{\text{eff}} = \frac{a+\mu-1}{2} + \frac{\Delta^2}{8} + \frac{\sigma^2}{8}. \tag{10}$$

We observe that the difference in the traffic of routes s and r is doubled under the full information mechanism (see (8) and (9)), where agents make routing decisions selfishly, compared to the socially efficient routing. This is an instance of the *tragedy of commons*, where each agent maximizes his own utility and does not take into account the congestion cost he imposes on the other agents on his route. Therefore, it may not be optimal for the principal to perfectly reveal his information about θ to the agents, as in the full information disclosure mechanism.

We now compare the optimal social welfare W^{eff} with the social welfare $W^{\text{no info}}$ under the no information disclosure mechanism. Under the no information disclosure mechanism, agents do not know θ and make their routing decisions only based on their ex-ante belief about θ (see (2) and (3)). Therefore, the social welfare under the no information disclosure mechanism, given by (4) is lower than the efficient social welfare because (i) the agents make their routing decisions selfishly, and (ii) the agents make their routing decisions without any knowledge about the realization of θ. The terms $\frac{\Delta^2}{8}$ and $\frac{\sigma^2}{8}$ in (10) capture the social welfare loss due to factors (i) and (ii) above, respectively.

In order to reduce the social welfare loss due to the agents' lack of information about θ, the principal may want to disclose information about the realization of θ to the agents. As discussed earlier, disclosing the realization of θ perfectly does not improve social welfare (see (4) and (7)). Therefore, the principal must utilize her superior information about route r's condition to strategically disclose information to the agents and influence their routing decision so as to improve the expected social welfare. This can be interpreted as providing *informational incentives* to the agents that align their objectives with that of the principal.

In the sequel, we explore various information disclosure mechanisms that the principal can employ to improve the expected social welfare. In Section IV, we explore public information disclosure mechanisms, where the principal reveals information about the realization of θ which is publicly observed by all agents. In Section V, we explore private information disclosure mechanisms, where the principal reveals information to each agent individually through a private communication channel.

IV. Public Information Disclosure

In this section, we consider mechanisms through which the principal reveals public information about the realization of θ to all agents. For instance, the principal can post traffic information on public road signs, or broadcast traffic updates through radio stations. Let $\mathcal{M}$ denote the set of all messages through which the principal can reveal information about the realization of θ. For instance, $\mathcal{M}$ can be the set of possible commute times on route r, or the number of congestion-causing accidents that have happened on route r. Given a message space $\mathcal{M}$, a public information disclosure mechanism can be fully described by $\psi : \Theta \to \Delta(\mathcal{M})$. For every realization of θ, ψ determines a probability distribution over the set of messages $\mathcal{M}$ that the principal sends. We note that the no information and full information disclosure mechanisms presented in Section III can be described as special instances of public information disclosure mechanisms by setting $\mathcal{M} = \emptyset$, and $\mathcal{M} = \Theta$ along with $\psi(\theta) = \theta$, respectively.

Given a public information disclosure mechanism $(\mathcal{M}, \psi)$, the agents update their belief about route r's condition θ after receiving a public message $m \in \mathcal{M}$ as,

$$\mathbb{P}\{\theta = \hat{\theta} | m\} = \frac{p_{\hat{\theta}} \psi(\hat{\theta})(m)}{\sum_{\tilde{\theta} \in \Theta} p_{\tilde{\theta}} \psi(\tilde{\theta})(m)}. \tag{11}$$

Using an argument similar to the one given in Section III-A, for every message realization $m \in \mathcal{M}$, the traffic at routes s and r are given by

$$x^{s,\text{public}}(m) = \frac{1}{2} + \frac{1}{2}\Delta_m, \tag{12}$$

$$x^{r,\text{public}}(m) = \frac{1}{2} - \frac{1}{2}\Delta_m, \tag{13}$$

where $\Delta_m := a - \mathbb{E}\{\theta|m\}$.

The principal's objective is to design a message space $\mathcal{M}$ along with a public information disclosure mechanism ψ so as to maximize the expected social welfare W. Formally,

$$\max_{\mathcal{M},\psi} \quad W$$

$$\text{subject to (12) and (13).}$$

Even though the principal can influence the agents' routing decisions for different realizations of θ by employing various public information disclosure mechanisms $(\mathcal{M}, \psi)$, we prove below that the expected social welfare W is independent of $(\mathcal{M}, \psi)$ for the model of Section II.

Theorem 1. *For every public information disclosure mechanisms $(\mathcal{M}, \psi)$, the expected social welfare W is given by $\frac{a+\mu-1}{2}$.*

The result of Theorem 1 states that the principal cannot benefit from employing a public information disclosure mechanism. We would like to note that for the model of Section II (i) the congestion functions $C^s(x^s)$ and $C^r(x^r)$ are linear in x^s and x^r, and (ii) the effect of route r's condition θ on the utility of an agent taking route r is linearly separable from the congestion cost $C^r(x^r)$. Because of features (i) and (ii), conditioned on the realization of message m, the expected social welfare is a linear function of Δ_m; this leads to the result of Theorem 1. In a model where either feature (i) or (ii) is absent, the result of Theorem 1 does not hold.

Remark 2. *Consider a model where the congestion costs $C^s(x^s)$ and $C^r(x^r)$ are nonlinear functions of x^s and x^r, respectively. Define a function $G : [a - \theta^M, a - \theta^1] \to [0, 1]$ as $G(\delta) := \{x : \ C^r(1-x) - C^s(x) = \delta\}$. Note that since $C^s(x^s)$ and $C^r(x^r)$ are strictly increasing in x^s and x^r, respectively, function $G(\delta)$ is well-defined. When $C^s(x^s) = x^s$ and $C^r(x^r) = x^r$, we have $G(\delta) = \frac{1}{2} + \frac{\delta}{2}$. Under a public information disclosure mechanism $(\mathcal{M}, \psi)$, conditioned on* *the realization of message* m*, the traffics at routes* s *and* r *are given by*

$$x^{s,public}(m) = G(\Delta_m), \tag{14}$$

$$x^{r,public}(m) = 1 - G(\Delta_m). \tag{15}$$

We can verify that if the function $C^s(G(\delta))$ *is convex (resp. concave) in* δ*, the optimal public information disclosure mechanism is the no information (resp. full information) mechanism.*[3] *In particular, if* $C^s(x^s)$ *and* $C^r(x^r)$ *are convex and concave (resp. concave and convex) in* x^s *and* x^r*, respectively, the function* $C^s(G(\delta))$ *is convex (resp. concave) in* δ*; thus, the optimal public information disclosure mechanism is the no information (resp. full information) mechanism. However, if the function* $C^s(G(\delta))$ *is neither convex nor concave in* δ*, there may exist instances of a set* Θ *of possible values for* θ*, along with a probability distribution over* Θ*, such that the optimal public information disclosure mechanisms is a public partial information disclosure mechanism.*

V. Private Information Disclosure

In this section, we study various private information disclosure mechanisms that the principal can use to reveal information about the realization of θ to the agents so as to improve the expected social welfare. For instance, the principal can provide individualized and private information to every agent through GPS-enabled devices such as routing suggestions in smart phone applications. Under a private information disclosure mechanism, the principal sends a private signal that is based on the realization of θ to every agent. Similar to a public information disclosure mechanism, the principal needs to determine (i) a set of messages (*i.e.* language) that he wants to use, and (ii) a mapping that determines for every realization of θ the probability according to which every signal is sent.

One class of private information disclosure mechanisms is the set of mechanisms where the principal sends to every agent a private and individualized routing recommendation (*i.e.* which route to take) based on the realization of θ. We refer to this subset of private information disclosure mechanisms as recommendation policies. We note that since the agents are strategic, they do not necessarily follow the principal's recommendation unless it is a best response for them. Using the revelation principle argument for information design problems (see [30]), we

[3]The result directly follows from an application of Jensen's inequality since $W = \mathbb{E}\{(a - G(\cdot)\}$.

can restrict attention, without loss of generality, to the set of recommendation policies where it is a best response for every agent to follow the recommendation he receives.

To avoid measure theoretic difficulties, we first assume that the principal sends $N > 0$ different recommendations to N groups of agents that have equal masses of $\frac{1}{N}$. We then consider the asymptotic case where $N \to \infty$.

Let $\sigma^N : \Theta \to \Delta(\{s, r\}^N)$ denote the recommendation policy that the principal employs for a given N. With some abuse of notation, let $\sigma^N(m^N|\theta)$ denote the probability that the principal sends routing recommendation $m^N := (m_1^N, ..., m_N^N) \in \{s, r\}^N$ to the N groups of agents, given that the state realization is $\theta \in \Theta$. Given a recommendation policy σ^N, each agent must be willing to take the recommended route given his information about route r's condition θ. This is captured by the following *obedience* condition for each agent belonging to group n, for $1 \leq n \leq N$,

(i) if $m_n^N = s$

$$\frac{1}{\sum_{\theta\in\Theta} p_\theta \sigma^N((s, m_{-n}^N)|\theta)} \sum_{\theta\in\Theta, m_{-n}^N \in\{s,r\}^{N-1}} p_\theta \sigma^N((s, m_{-n}^N)|\theta)(a - \frac{1}{N}\sum_{1\leq i\leq N} \mathbf{1}_{\{m_i^N = s\}})$$
$$\geq$$
$$\frac{1}{\sum_{\theta\in\Theta} p_\theta \sigma^N((s, m_{-n}^N)|\theta)} \sum_{\theta\in\Theta, m_{-n}^N \in\{s,r\}^{N-1}} p_\theta \sigma^N((s, m_{-n}^N)|\theta)(\theta - \frac{1}{N}\sum_{1\leq i\leq N} \mathbf{1}_{\{m_i^N = r\}}), \quad (16)$$

(ii) if $m_n^N = r$

$$\frac{1}{\sum_{\theta\in\Theta} p_\theta \sigma^N((r, m_{-n}^N)|\theta)} \sum_{\theta\in\Theta, m_{-n}^N \in\{s,r\}^{N-1}} p_\theta \sigma^N((r, m_{-n}^N)|\theta)(\theta - \frac{1}{N}\sum_{1\leq i\leq N} \mathbf{1}_{\{m_i^N = r\}})$$
$$\geq$$
$$\frac{1}{\sum_{\theta\in\Theta} p_\theta \sigma^N((r, m_{-n}^N)|\theta)} \sum_{\theta\in\Theta, m_{-n}^N \in\{s,r\}^{N-1}} p_\theta \sigma^N((r, m_{-n}^N)|\theta)(a - \frac{1}{N}\sum_{1\leq i\leq N} \mathbf{1}_{\{m_i^N = s\}}). \quad (17)$$

The above obedience constraints are the analogue of the *incentive compatibility* constraints in mechanism design problems, and can be interpreted similarly as follows. The left hand side of condition (16) (resp. (17)) expresses the expected utility of an agent in group n, $1 \leq n \leq N$, if he follows the recommendation to take route s (resp. r) given his ex-post belief about θ after he receives the recommendation, assuming that the other agents are following their recommendations. The right hand side of condition (16) (resp. (17)) expresses the expected utility of an agent in group n, if he deviates from his recommendation and takes route r (resp. s) instead of s (resp. r) given his ex-post belief about θ. The obedience constraint (16) therefore requires that it is a best response for every agent to follow the recommendation, given his ex-post belief about θ, assuming that other agents follow their routing recommendations. We note that unlike standard mechanism design problems, there is no individual rationality constraint, since an agent can simply ignore the recommendation and choose any route he wishes.

Let $x^{s,N}(\theta) \in \{\frac{1}{N}, \frac{2}{N}, \ldots, \frac{N}{N}\}$ denote the mass of agents that take route s when the state is θ under σ^N. Note that the set of obedience constraints (16) and (17) are linear in $\sigma^N(\cdot|\cdot)$ and identical for all N groups of agents. Therefore, by symmetry, we can restrict attention to the set of recommendation policies for the principal where she selects $N \cdot x^{s,N}(\theta)$ groups randomly, recommends to them to take route s, and recommends to the agents in the remaining groups to take route r.

Therefore, for $N \to \infty$ the set of recommendation policies for the principal can be characterized by $y(\theta) \in [0, 1]$, where $y(\theta)$ denotes the mass of agents receiving the recommendation to take route s, *i.e.* $x^s(\theta) = y(\theta)$ and $x^r(\theta) = 1 - y(\theta)$. When the state is θ, the principal recommends route s (resp. r) to every agent with probability $y(\theta)$ (resp. $1 - y(\theta)$) independent of her recommendation to other agents.

Under the information policy σ, let $U^\sigma(s, \theta) := a - y(\theta)$ and $U^\sigma(r, \theta) := \theta - (1 - y(\theta))$ denote an agent's utility from taking routes s and r, respectively, when route r's condition is θ. The set of obedience constraints (16) and (17) for each agent can be then written as

$$\sum_{\theta \in \Theta} p_\theta y(\theta) U^\sigma(s, \theta) \geq \sum_{\theta \in \Theta} p_\theta y(\theta) U^\sigma(r, \theta), \tag{18}$$

$$\sum_{\theta \in \Theta} p_\theta (1 - y(\theta)) U^\sigma(r, \theta) \geq \sum_{\theta \in \Theta} p_\theta (1 - y(\theta)) U^\sigma(s, \theta). \tag{19}$$

Therefore, the problem that the principal faces is to determine a recommendation policy that maximizes the expected social welfare subject to the obedience constraints above; this optimization problem is given by[4]

$$\max_{\{y(\theta),\theta\in\Theta\}} W$$

$$\text{subject to } (18) \text{ and } (19).$$

A. Implementable Outcomes

To determine an optimal recommendation policy, we first specify the set of feasible routing outcomes/recommendation policies that satisfy the obedience constraints (18) and (19).

Lemma 1. *A routing outcome* $\{x^s(\theta), x^r(\theta), x^s(\theta)+x^r(\theta)=1,\ \theta\in\Theta\}$ *is implementable if and only if*

$$\mathbb{E}\left\{x^s(\theta)\left[\left(\frac{1}{2}+\frac{\Delta_\theta}{2}\right)-x^s(\theta)\right]\right\}\geq 0, \tag{20}$$

$$\mathbb{E}\left\{x^r(\theta)\left[\left(\frac{1}{2}+\frac{\Delta_\theta}{2}\right)-x^r(\theta)\right]\right\}\geq 0. \tag{21}$$

We note that the outcomes under the no information and full information disclosure policies, given by (2)-(3) and (5)-(6), respectively, satisfy conditions (20) and (21) with equality. That is, they are the corner points of the set of implementable outcomes. The set of implementable outcomes is depicted in Figure 2 for an example with $|\Theta|=2$.

B. Incentivizing the Socially Efficient Routing

Using the result of Lemma 1, we can determine the necessary and sufficient condition to implement the efficient allocation $\{x^{s,\text{eff}}(\theta), x^{r,\text{eff}}(\theta), \theta\in\Theta\}$ through the recommendation policy below.

Theorem 2. *The efficient routing policy* x^{eff} *is implementable through an information disclosure policy if and only if*

$$\sigma^2 \geq 2|\Delta| - \Delta^2. \tag{22}$$

[4]We note that we can restrict attention, without loss of optimality, to policies where $y(\theta)$ is deterministic. This is because the set of obedience constraints only depends on the expected value of $y(\theta)$. Moreover, the principal's objective is a concave function of $y(\theta)$ (see (1)). Thus, by the Jensen's inequality, an optimal recommendation policy is a recommendation policy where $y(\theta)$ is deterministic for every $\theta\in\Theta$.

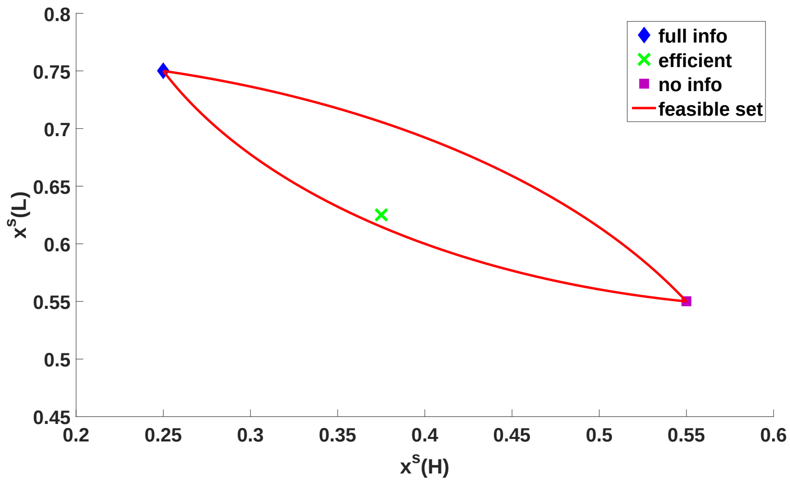


Fig. 2: The set of implementable outcomes for $a = 2$, $\Theta = \{L, H\}$, $L = 1.5$, $H = 2.5$, $p_L = 0.6$, and $p_H = 0.4$.

We note that $|\Delta| = \frac{|a-\mu|}{m} \leq 1$ by Assumption 1; thus, $2|\Delta| - \Delta^2 \geq 0$. For ex-ante symmetric routes (*i.e.* $\mu = a$), we have $\Delta = 0$, and the efficient outcome is always implementable for any distribution of θ. However, if the two routes are ex-ante asymmetric (*i.e.* $\mu \neq a$), to incentivize the efficient policy, the variance of θ must be greater than the threshold (22), which depends on the expected difference between the routes. We further elaborate on this issue below.

As we discussed above, we can view the routing recommendation by the principal to the agents as an informational incentive that she provides so as to influence the routing decision of each agent. When the routes are symmetric, *i.e.* $\Delta = 0$, under the no information disclosure policy, each agent (at equilibrium) is indifferent between taking either of the routes; see (2) and (3). Therefore, the principal can persuade (*i.e.* recommend to) an agent to take a specific route even when she does not have significant information superiority over him (*i.e.* σ^2 is small). However, when the routes are asymmetric, *i.e.* $\Delta \neq 0$, under the no information disclosure policy, each agent has a strict preference over route s (resp. r) if $\Delta > 0$ (resp. $\Delta < 0$). Thus, the principal needs a strictly positive incentive to persuade an agent to take the route that is not aligned with his original preference. This implies that the information the principal holds must be valuable enough to enable her to offer adequate informational incentives to persuade an agent to follow her recommendation. Condition (22) captures the value of the principal's information about θ in terms of σ^2.

Figure 3 depicts the maximum expected social welfare the principal can achieve for different combinations of σ^2 and Δ by utilizing a recommendation policy in an example with $|\Theta| = 2$. We note that for pairs (σ, Δ) that satisfy condition (22) of Theorem 2, the principal can implement the socially efficient outcomes. However, when this condition is violated, the performance of the best outcome decreases.

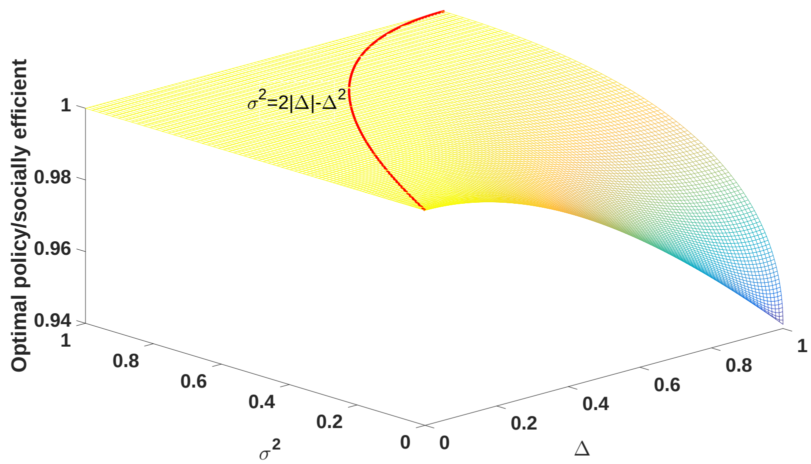


Fig. 3: The best implementable outcomes with respect to the socially efficient outcome for $a = 2$, $\Theta = \{L, H\}$, and $p_L = p_H = 0.5$.

Remark 3. *A result similar to that of Theorem 2 can be obtained for general congestion functions $C^r(x^r)$ and $C^s(x^s)$, where the condition that is the analogue of (22) depends on higher order moments of θ.*

VI. Dynamic Setting

In this section, we study a dynamic setting with time horizon $T = 2$, *i.e* $t \in \{1, 2\}$, where route r's condition θ_t, $t = 1, 2$, has uncontrolled Markovian dynamics with transition probability $\mathbf{P} \in \mathbb{R}^{|\Theta|\times|\Theta|}$. We assume that $\mathbf{P}[p_{\theta^1}, \dots, p_{\theta^M}]^T = [p_{\theta^1}, \dots, p_{\theta^M}]^T$, that is, the marginal probability distribution of θ_2 is the same that of θ_1.

We consider a situation where the same group of agents commute from the origin to the destination every day. Therefore, agents at $t = 2$ may have learnt new information from their observations at $t = 1$. We study the problem of designing an optimal dynamic private information disclosure policy by the principal. The investigation of this two-step dynamic mechanism provides some insight into how the results for a static setting change in a dynamic setting where agents can learn from their past experience. We note that by the result of Theorem 1, the study of dynamic public information disclosure mechanisms in a dynamic setting does not introduce any issue in addition to those present in the study of static mechanisms within the context of the model of Section II.

We consider three scenarios depending on the agents' observations at $t = 1$ as follows: (i) agents do not make any environmental observations (*i.e.,* neither the condition of the risky route nor the traffic (*i.e.* mass of agents/cars) on routes s and r, (ii) agents who take route r observe only its condition θ_1, and (iii) each agent observes only the traffic on the route he takes at $t = 1$. In a real world situation, the agents can have noisy observations of θ_1 as well as a noisy observation of the number of cars traveling the route. Therefore, the study of the three scenarios described above will allow us to understand the effect of each type of learning (piece of information) on the solution of the dynamic problem and uncover its qualitative properties.

In all of these scenarios, we assume that the principal's routing recommendation policy at $t = 2$ does not depend on the agent's decisions at $t = 1$. We make this assumption for the following reasons. (1) If the principal wants to incorporate the agents' past decisions into her routing recommendation policy, she needs to monitor every agent's location over time; this may not be feasible due to technological limitations and/or privacy concerns. (2) If the principal can incorporate the agents' past decisions into her routing recommendation policy, then her optimal strategy would be to not disclose any further information to every agent that does not follow her routing recommendation (*i.e.* punish him). On one hand, such a punishment scheme may not be desirable in practical settings. On the other hand, if such a punishment scheme is permitted, then the principal can incentivize any desired routing behavior in a dynamic setting with long enough horizon if the agents are sufficiently patient and θ_t does not have deterministic dynamics (*i.e.* the principal has information superiority over the agents for all times). Therefore, in the sequel we restrict attention to the class of dynamic recommendation policies where the principal does not observe/incorporate the agents' decisions at $t = 1$ when designing her policy.

As a result, the set of recommendation policies for the principal can be characterized by $\sigma := \{y_1(\theta_1), y_2^s(\theta_2, \theta_1), y_2^r(\theta_2, \theta_1),\ \forall \theta_1, \theta_2 \in \Theta\}$. The principal's routing policy at $t = 1$ is given by $y_1(\theta)$ (resp. $1 - y_1(\theta)$), which denotes the probability that route s (resp. r) is recommended when route r's condition is θ_1. For $t = 2$, $y_2^s(\theta_2, \theta_1)$ (resp. $y_2^r(\theta_2, \theta_1)$) denotes the probability that route s is recommended to agents who took route s (resp. r) at $t = 1$, when route r's condition at $t = 1$ and $t = 2$ are θ_1 and θ_2, respectively. Similarly, $1 - y_2^s(\theta_2, \theta_1)$ (resp. $1 - y_2^r(\theta_2, \theta_1)$) denotes the probability that route r is recommended to agents who took route s (resp. r) at

$t = 1$, when route r's condition at $t = 1$ and $t = 2$ are θ_1 and θ_2, respectively.

A. Case (i): No Environmental Observations

Consider a situation where the agents do not make any observations about the road condition θ_1 and/or the number of agents at the route they took at $t = 1$. Therefore, the agents infer information about θ_1 only based on the recommendation that they receive at $t = 1$.

Let

$$U_2^\sigma(s, \theta_2, \theta_1) := a - y_1(\theta_1) y_2^s(\theta_2, \theta_1) - (1 - y_1(\theta_1)) y_2^r(\theta_2, \theta_1),$$

$$U_2^\sigma(r, \theta_2, \theta_1) := \theta_2 - y_1(\theta_1)(1 - y_2^s(\theta_2, \theta_1)) - (1 - y_1(\theta_1))(1 - y_2^r(\theta_2, \theta_1)),$$

denote the utility of routes s and r at $t = 2$ given that all agents follow their recommendations. Moreover, define,

$$\mathbb{P}^\sigma\{\theta_2, \theta_1, s, s\} = y_2^s(\theta_2, \theta_1) y_1(\theta_1) \mathbf{P}(\theta_2, \theta_1) p_{\theta_1},$$

$$\mathbb{P}^\sigma\{\theta_2, \theta_1, s, r\} = y_2^r(\theta_2, \theta_1)(1 - y_1(\theta_1)) \mathbf{P}(\theta_2, \theta_1) p_{\theta_1},$$

$$\mathbb{P}^\sigma\{\theta_2, \theta_1, r, s\} = (1 - y_2^s(\theta_2, \theta_1)) y_1(\theta_1) \mathbf{P}(\theta_2, \theta_1) p_{\theta_1},$$

$$\mathbb{P}^\sigma\{\theta_2, \theta_1, r, r\} = (1 - y_2^r(\theta_2, \theta_1))(1 - y_1(\theta_1)) \mathbf{P}(\theta_2, \theta_1) p_{\theta_1}.$$

Then the set of obedience constraints for $t = 2$ are as follows:

(a) Recommendation s at $t = 2$ and s at $t = 1$:

$$\sum_{\theta_1, \theta_2 \in \Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, s, s\} U_2(s, \theta_2, \theta_1) \geq \sum_{\theta_1, \theta_2 \in \Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, s, s\} U_2(r, \theta_2, \theta_1). \tag{23}$$

(b) Recommendation s at $t = 2$ and r at $t = 1$:

$$\sum_{\theta_1, \theta_2 \in \Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, s, r\} U_2(s, \theta_2, \theta_1) \geq \sum_{\theta_1, \theta_2 \in \Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, s, r\} U_2(r, \theta_2, \theta_1) \tag{24}$$

(c) Recommendation r at $t = 2$ and s at $t = 1$:

$$\sum_{\theta_1, \theta_2 \in \Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, r, s\} U_2(r, \theta_2, \theta_1) \geq \sum_{\theta_1, \theta_2 \in \Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, r, s\} U_2(s, \theta_2, \theta_1) \tag{25}$$

(d) Recommendation r at $t = 2$ and r at $t = 1$:

$$\sum_{\theta_1,\theta_2\in\Theta} \mathbb{P}^\sigma\{\theta_2,\theta_1,r,r\}U_2(s,\theta_2,\theta_1) \geq \sum_{\theta_1,\theta_2\in\Theta} \mathbb{P}^\sigma\{\theta_2,\theta_1,r,r\}U_2(r,\theta_2,\theta_1) \tag{26}$$

We note that since the agents' observations at $t = 1$ include only the routing recommendation they receive at that time, the agents act myopically at $t = 1$, as they cannot learn additional information by themselves. Thus, the set of obedience constraints at $t = 1$ are the same as those of the static problem; these constraints are given by

$$\sum_{\theta_1\in\Theta} p_\theta y_1(\theta_1)U_1^\sigma(s,\theta_1) \geq \sum_{\theta_1\in\Theta} p_\theta y_1(\theta_1)U_1^\sigma(r,\theta_1), \tag{27}$$

$$\sum_{\theta_1\in\Theta} p_\theta(1-y_1(\theta_1))U_1^\sigma(r,\theta_1) \geq \sum_{\theta_!\in\Theta} p_\theta(1-y_1(\theta_1))U_1^\sigma(s,\theta_1), \tag{28}$$

where $U_1^\sigma(s,\theta_1) = a - y_1(\theta_1)$ and $U_1^\sigma(r,\theta_1) = \theta_1 - (1 - y_1(\theta_1))$.

The expected social welfares at $t = 1$ and $t = 2$ are given by

$$W_1 := \sum_{\theta_1\in\Theta} p_{\theta_1}\left[y(\theta_1)U_1^\sigma(s,\theta_1) + (1-y(\theta_1))U_1^\sigma(r,\theta_1)\right], \tag{29}$$

$$\begin{aligned} W_2 := \sum_{\theta_1,\theta_2\in\Theta} p_{\theta_1}\mathbf{P}(\theta_2,\theta_1)\Big[&\big[y(\theta_1)y_2^s(\theta_2,\theta_1) + (1-y(\theta_1))y^r(\theta_2,\theta_1)\big]U_1^\sigma(s,\theta_2,\theta_1) \\ &+ \big[y(\theta_1)(1-y_2^s(\theta_2,\theta_1)) + (1-y(\theta_1))(1-y_2^r(\theta_2,\theta_1))\big]U_2^\sigma(r,\theta_2,\theta_1)\Big] \end{aligned} \tag{30}$$

Therefore, the optimal routing recommendation policy by the principal when the agents do not have any environmental observations is given by the solution to the following optimization problem

$$\begin{aligned} &\max_{\sigma} \quad W_1 + W_2 \\ &\text{subject to } (23) - (28). \end{aligned}$$

In this paper, we do not provide a closed solution to the above maximization problem for a general transition matrix $\mathbf{P}$. Nevertheless, we consider two special cases below: (a) when θ_2 is identically distributed and independent of θ_1 (i.e. no correlation), and (b) when θ_2 is identical to θ_1 (*i.e.* perfect correlation). We argue below that that for special cases (a) and (b) the performance of an optimal dynamic recommendation policy per time step is equal to that of optimal static recommendation policy.

For case (a), it is easy to verify that the repetition of the optimal static recommendation policy is an optimal dynamic recommendation policy. For case (b), at $t = 1$, the optimal static routing policy is implementable since the obedience constraints at $t = 1$ are identical to those in the static problem, At $t = 2$, consider the recommendation policy $y^s(\theta_2) = 1$ and $y^r(\theta_2) = 0$. That is, at $t = 2$, the principal recommends to every agent to take the same route he took at $t = 1$. If an agent is willing to follow his recommendation at $t = 1$, then he is also willing to take the exact route for the next day, since he does not learn any new information after $t = 1$, and route r's condition remains the same. Therefore, $y^s(\theta_2) = 1$ and $y^r(\theta_2) = 0$ is implementable at $t = 2$. It is easy to verify that the performance of the dynamic recommendation policy described above is identical to that of the optimal static recommendation policy.

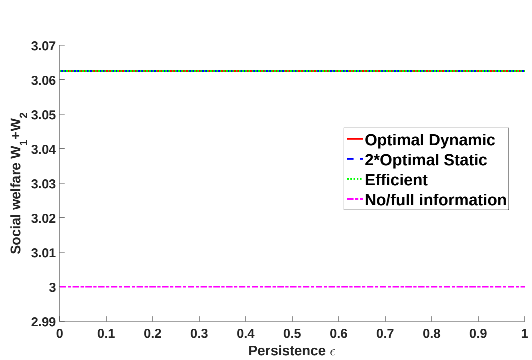
Given a general transition matrix $\mathbf{P}$, every agent forms an updated belief about θ_1 after receiving his recommendation at $t = 1$. As the correlation between θ_1 and θ_2 increases, the information that the agent learns at $t = 1$ becomes more valuable to him, and consequently, the principal's information superiority decreases. The argument given above for case (b) states that even when $\theta_1 = \theta_2$, the principal can achieve the same expected social welfare at $t = 2$ as in the static setting. Therefore, we conjecture that the (partial) results we proved above for the special cases (a) and (b), hold in general for any correlation between 0 and 1.

Conjecture 1. *For every transition matrix* $\mathbf{P}$*, the performance of an optimal dynamic recommendation policy per time step is equal to that of an optimal static recommendation policy.*

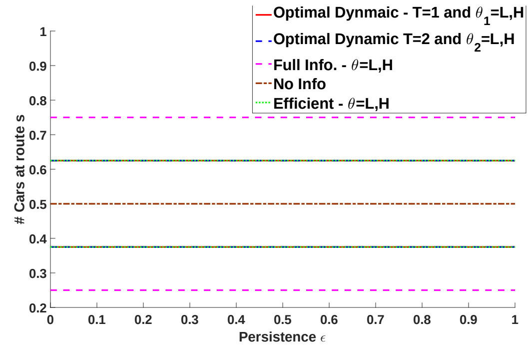
We examine the above conjecture through a numerical simulation below. Consider a setting where $\Theta = \{L, H\}$ with $p_L = p_H = 0.5$. Assume that the transition matrix $\mathbf{P}$ is given by

$$\mathbf{P} := \begin{pmatrix} p_L + \frac{\epsilon}{2} & p_L - \frac{\epsilon}{2} \\ p_H - \frac{\epsilon}{2} & p_H + \frac{\epsilon}{2} \end{pmatrix}, \tag{31}$$

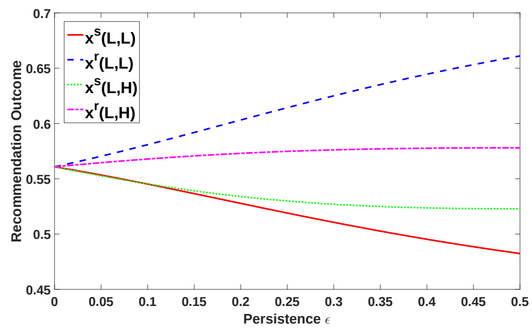
where $\epsilon \in [0, 1]$ denotes the persistence (i.e. correlation) of route r's condition from $t = 1$ to $t = 2$. Figures 4-6 depict the optimal dynamic recommendation policies vs. different values of ϵ for three pairs of σ^2 and Δ. As seen in Fig. 4a-6a, in all three examples the performance of the dynamic recommendation policy per time step is the same as in the optimal static recommendation policy for different values of ϵ. Moreover, the total number of cars at each route at $t = 1, 2$ are identical under the optimal dynamic recommendation policy; see Figures 4b-6b.



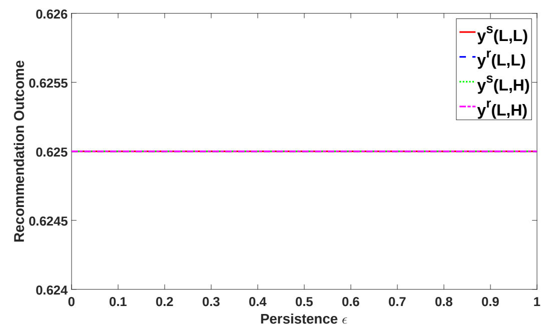
(a) The social welfare



(b) The number of agents at routes at $t = 1, 2$ for different route r's condition



(c) The recommendation outcome at $t = 2$ for $\theta_1 = L$

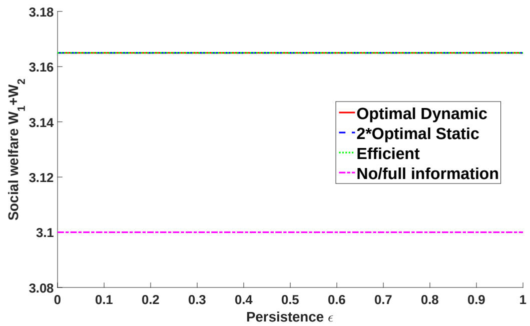


(d) The recommendation outcome at $t = 2$ for $\theta_1 = H$

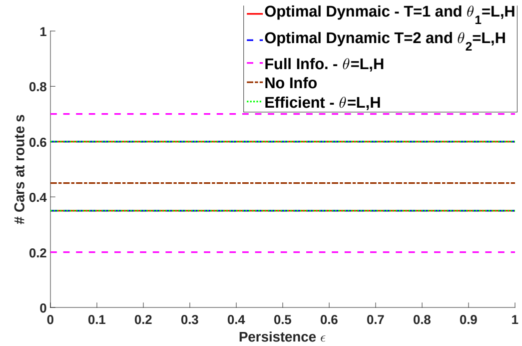
Fig. 4: Case (i) - the optimal dynamic recommendation policy for different values of persistence ϵ, for $\theta \in \{L, H\}$, $L = 1.5$, $H = 2.5$, $p_L = p_H = 0.5$ ($\Delta = 0$, $\sigma^2 = 0.25$)

In the first two examples (see Figures 4 and 5), the pair (σ^2, Δ) satisfies condition (22) of Theorem 2; thus, the performance of the optimal dynamic and static recommendation policies are the same as the efficient social welfare. However, in the third example (see Figure 6), where the pair (σ^2, Δ) does not satisfy condition (22), the performance of the optimal dynamic and static recommendation policies are inferior to that of the social welfare maximizing policy.

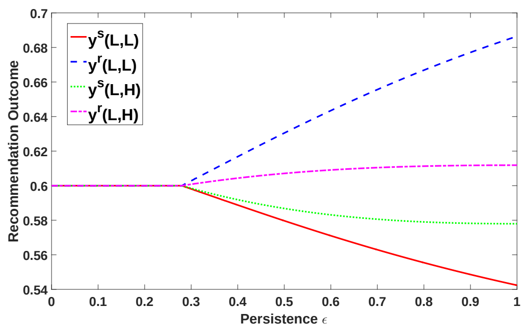
In Section V-B, we argued that σ^2 represents a measure of the principal's power in terms of the informational incentives she can provide to the agents. Moreover, we argued that Δ indicates the agents' ex-ante preference towards one of the routes. Accordingly, we interpreted the condition of Theorem 2 as requiring that the principal's informational power to be greater than the agents' ex-ante preference towards one of the routes. A similar interpretation can be given here by comparing the recommendation outcomes for different pairs of (σ^2, Δ). As seen in Figures 4c and 4d, when the agents do not have an ex-ante preference towards either of the routes (*i.e.* $\Delta = 0$), the optimal recommendation policy prescribes the same routing suggestion



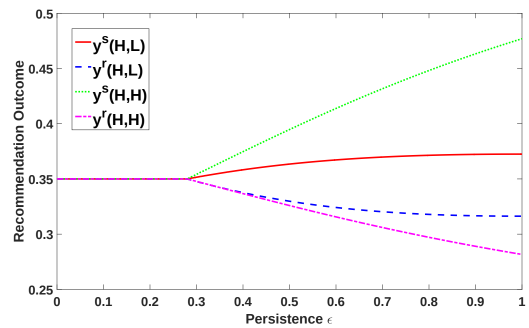
(a) The social welfare



(b) The number of agents at routes at $t = 1, 2$ for different route r's condition



(c) The recommendation outcome at $t = 2$ for $\theta_1 = L$



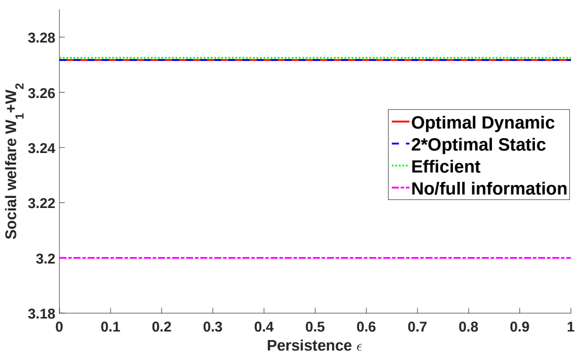
(d) The recommendation outcome at $t = 2$ for $\theta_1 = H$

Fig. 5: Case (i) - the optimal dynamic recommendation policy for different values of persistence ϵ, for $\theta \in \{L, H\}$, $L = 1.6$, $H = 2.6$, $p_L = p_H = 0.5$ ($\Delta = 0.1$, $\sigma^2 = 0.25$)

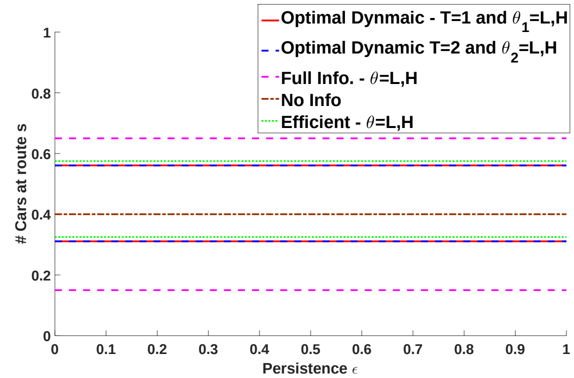
for all groups of agents at $t = 2$ irrespective of what they have learnt at $t = 1$. As the agents develop an ex-ante preference towards one of the routes (see Figure 5 where $\Delta = 0.1$), the optimal recommendation policy prescribes the same routing suggestion for low values of ϵ; but as ϵ increases, the optimal recommendation policy prescribes different routing suggestions depending on what every agent has learnt at $t = 1$. When agents have a high ex-ante preference towards one of the routes (see Figure 5 where $\Delta = 0.2$), the optimal recommendation policy prescribes different routing suggestions for every value of $\epsilon \neq 0$ depending on what every agent has learnt at $t = 1$.

B. *Case (ii): Observing* θ

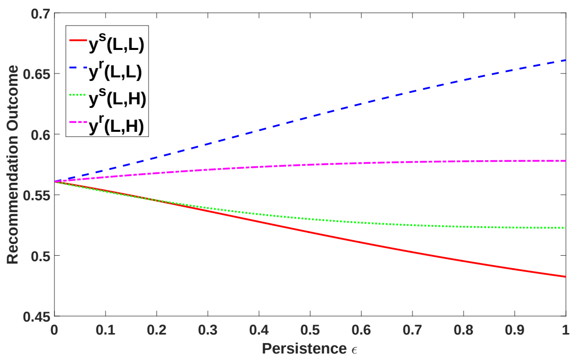
Consider a situation where agents who take route r at $t = 1$ observe θ_1 perfectly. Therefore, at $t = 2$ agents have heterogeneous/asymmetric information about θ_2 depending on which route they took at $t = 1$.



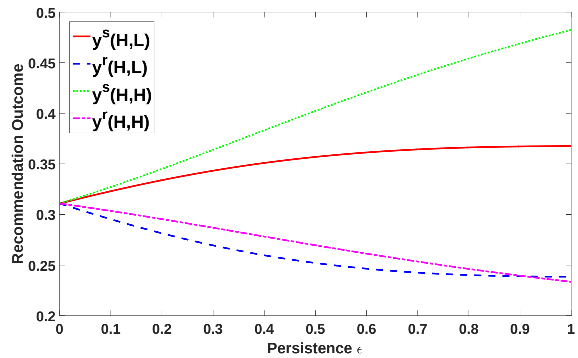
(a) The social welfare



(b) The number of agents at routes at $t = 1, 2$ for different route r's condition



(c) The recommendation outcome at $t = 2$ for $\theta_1 = L$



(d) The recommendation outcome at $t = 2$ for $\theta_1 = H$

Fig. 6: Case (i) - the optimal dynamic recommendation policy for different values of persistence ϵ, for $\theta \in \{L, H\}$, $L = 1.7$, $H = 2.7$, $p_L = p_H = 0.5$ ($\Delta = 0.2$, $\sigma^2 = 0.25$)

As a result, the set of obedience constraints at $t = 2$ is as follows:

(a) Recommendation s at $t = 2$ and s at $t = 1$:

$$\sum_{\theta_1,\theta_2\in\Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, s, s\} U_2(s, \theta_2, \theta_1) \geq \sum_{\theta_1,\theta_2\in\Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, s, s\} U_2(r, \theta_2, \theta_1). \tag{32}$$

(b) Recommendation s at $t = 2$ and r at $t = 1$: for every $\theta_1 \in \Theta$

$$\sum_{\theta_2\in\Theta} \mathbb{P}^\sigma\{\theta_2, s, r\} U_2(s, \theta_2, \theta_1) \geq \sum_{\theta_2\in\Theta} \mathbb{P}^\sigma\{\theta_2, s, r\} U_2(r, \theta_2, \theta_1) \tag{33}$$

(c) Recommendation r at $t = 2$ and s at $t = 1$:

$$\sum_{\theta_1,\theta_2\in\Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, r, s\} U_2(r, \theta_2, \theta_1) \geq \sum_{\theta_1,\theta_2\in\Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, r, s\} U_2(s, \theta_2, \theta_1) \tag{34}$$

(d) Recommendation r at $t=2$ and r at $t=1$: for every $\theta_1 \in \Theta$

$$\sum_{\theta_2 \in \Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, r, r\} U_2(r, \theta_2, \theta_1) \geq \sum_{\theta_2 \in \Theta} \mathbb{P}^\sigma\{\theta_2, \theta_1, r, r\} U_2(s, \theta_2, \theta_1) \tag{35}$$

We note that the obedience constraints (32) and (34) are the same as (23) and (25). This is because if an agent takes route s at $t=1$, his only new information at $t=2$ is the routing recommendation that he receives at $t=1$, and the situation at $t=2$ is similar to the scenario with no environmental observations in Section VI-A. However, if an agent takes route r at $t=1$, he observes θ_1 perfectly. Therefore, for every possible value of θ_1 at $t=1$, there exists a corresponding obedience constraint at $t=2$ expressed (33) and (35).

Since agents can learn/observe θ_1 at $t=1$ by taking route r, the agents' incentives to follow the principal's routing recommendation are different from those in a static setting. An agent may want to deviate from the recommendation to take route s and instead take route r so as to observe θ_1; he can then utilize this observation to his benefit, as he now has better information about route r's condition at $t=2$. In other words, an agent can coordinate his routing decision at $t=2$ with his routing decision at $t=1$. Therefore, the set of obedience constraints at $t=1$ must consider all possible future plans that an agent can utilize at $t=2$ after his deviation at $t=1$; this set of obedience constraints can be described as follows:

(a) Recommendation s at $t=1$: for every $\lambda : \{s,r\} \times \Theta \rightarrow \{s,r\}$

$$\begin{aligned}
&\sum_{\theta_1 \in \Theta} p_{\theta_1} y(\theta_1) \Big[U_1^\sigma(s, \theta_1) + \sum_{\theta_2 \in \Theta} \mathbf{P}(\theta_2, \theta_1) \Big(y_2^s(\theta_2, \theta_1) U_2^\sigma(s, \theta_2, \theta_1). \\
&\qquad\qquad + (1 - y_2^s(\theta_2, \theta_1)) U_2^\sigma(r, \theta_2, \theta_1) \Big) \Big] \\
&\geq \\
&\sum_{\theta_1 \in \Theta} p_{\theta_1} y(\theta_1) \Big[U_1^\sigma(r, \theta_1) + \sum_{\theta_2 \in \Theta} \mathbf{P}(\theta_2, \theta_1) \Big(y_2^s(\theta_2, \theta_1) U_2^\sigma(\lambda(s, \theta_1), \theta_2, \theta_1) \\
&\qquad\qquad + (1 - y_2^t(\theta_2, \theta_1)) U_2^\sigma(\lambda(r, \theta_1), \theta_2, \theta_1) \Big) \Big],
\end{aligned} \tag{36}$$

(b) Recommendation r at $t=1$: for every $\lambda : \{s,r\} \to \{s,r\}$

$$\sum_{\theta_1 \in \Theta} p_{\theta_1}(1-y(\theta_1)) \Bigg[U_1^{\sigma}(r,\theta_1) + \sum_{\theta_2 \in \Theta} \mathbf{P}(\theta_2,\theta_1) \big(y_2^r(\theta_2,\theta_1) U_2^{\sigma}(s,\theta_2,\theta_1). \\ +(1-y_2^r(\theta_2,\theta_1)) U_2^{\sigma}(r,\theta_2,\theta_1) \big) \Bigg] \\ \geq \\ \sum_{\theta_1 \in \Theta} p_{\theta_1}(1-y(\theta_1)) \Bigg[U_1^{\sigma}(s,\theta_1) + \sum_{\theta_2 \in \Theta} \mathbf{P}(\theta_2,\theta_1) \Big(y_2^r(\theta_2,\theta_1) U_2^{\sigma}(\lambda(s),\theta_2,\theta_1) \\ +(1-y_2^r(\theta_2,\theta_1)) U_2^{\sigma}(\lambda(r),\theta_2,\theta_1) \Big) \Bigg]. \quad (37)$$

In the obedience constraints above, an agent's plan at $t=2$ after his deviation at $t=1$ is denoted by λ. If an agent deviates from a recommendation to take route s at $t=1$ and takes route r instead, his plan λ at $t=2$ depends on his observation of θ_1 as well as his routing recommendation at $t=2$ (see (36)). If an agent deviates from a recommendation to take route r at $t=1$ and instead takes route s, his plan λ at $t=2$ depends only his routing recommendation at $t=2$ since he does not observe θ_1 at $t=1$ (see (37)).

Consequently, the optimal routing recommendation policy by the principal, when agents taking route r at $t=1$ observe θ_1, is given by the solution to the following optimization problem

$$\max_{\sigma} \quad W_1 + W_2 \\ \text{subject to } (32)-(37).$$

The above optimization problem has $6+2|\Theta|+2^{2|\Theta|}$ number of constraints, which grows exponentially in $|\Theta|$, making it difficult to provide a closed form solution to the above problem in general. Therefore, we investigate the properties of an optimal dynamic recommendation policy, when agents can observe θ_1, through numerical simulations below.

Consider a setting similar to the one in Section VI-A, where $\Theta = \{L,H\}$ with $p_L = p_H = 0.5$ and the transition matrix $\mathbf{P}$ is given by (38). Figures 7-8 depict the optimal dynamic recommendation policies vs. different values of ϵ for two pairs of σ^2 and Δ. As seen in Figures 7a-8a, in both examples the performance of the dynamic recommendation policy is decreasing in ϵ.

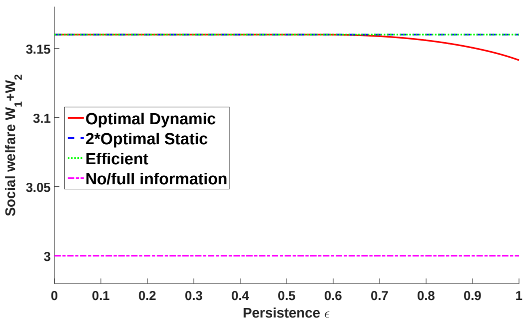
In the first example (see Figure 7)), the pair (σ^2, Δ) satisfies condition (22) of Theorem

2. Therefore, for low values of ϵ, where the information the agents learn at $t = 1$ does not reduce the principal's information superiority to the point where condition (22) is not satisfied at $t = 2$, the principal can implement the efficient routing policy (see Figure 7a)). However, as ϵ increases, the principal cannot implement the efficient routing at $t = 1, 2$. As a result, the optimal recommendation policy is different from the efficient routing policy for higher values of ϵ (see Figure 7b)). Moreover, for higher values of ϵ the optimal recommendation policy at $t = 2$ depends on the route an agent took at $t = 1$ (see Figures 7c and 7d). We note that for $\epsilon = 1$, $y^r(L, L) = 1$ and $y^r(H, H) = 0$. This is because, when $\epsilon = 1$, an agent who takes route r at $t = 1$ perfectly knows θ_2 since $\theta_2 = \theta_1$. Therefore, an agent who takes route r (*i.e.* observes θ_1) at $t = 1$ always chooses the route with the better condition at $t = 2$.

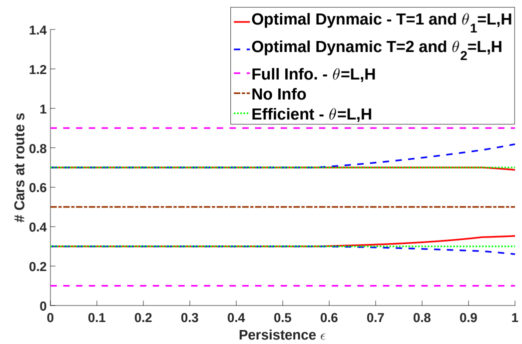
In the second example (see Figure 8), the pair (σ^2, Δ) does not satisfy condition (22) of Theorem 2. Therefore, the performance of the optimal dynamic recommendation policy is *strictly* decreasing in ϵ for all values of ϵ (see Figure 8a). Moreover, the optimal recommendation policy at $t = 1, 2$ is always different from the efficient routing policy (see Figure 8b). As seen in Figures 8c and 8d, for all $\epsilon \neq 0$, the optimal recommendation policy at $t = 2$ depends on which route an agent took at $t = 1$. We note that when $\epsilon = 1$, the optimal routing recommendation policy at $t = 2$ results in the same traffic on the routes as the one under the full information disclosure mechanisms (see Figure 8b). That is, for very high values of ϵ, where the agents have considerable incentive to experiment at $t = 1$ by taking route r, the principal promises to perfectly disclose his information at $t = 2$ so that the agents are willing to follow her recommendation at $t = 1$. As a consequence, in contrast to Figures 7c and 7d, in Figures 8c and 8d we have $y^r(L, L) \neq 1$ and $y^r(H, H) \neq 0$ for $\epsilon = 1$. This is because under the optimal recommendation policy the utility from taking either route is the same when $\epsilon = 1$, and thus, an agent is indifferent between them even though he knows θ_2 perfectly.

C. Case (iii): Observing the Traffic

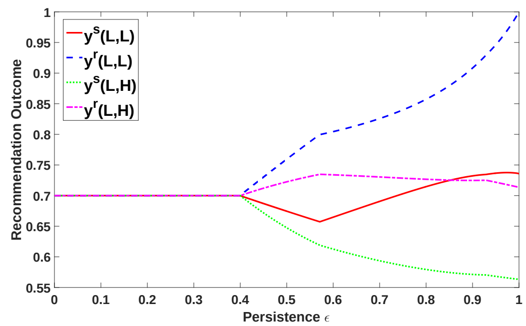
Consider a situation where each agent observes the traffic on the route he has taken at $t = 1$, , given by $y_1(\theta_1)$ or $1 - y_1(\theta_1)$. Since we assume that there exists a unit mass of agents, which is common knowledge among all agents, each agent can determine the traffic at $t = 1$ on both routes, and thus, all agents have identical information at $t = 2$. Consequently, the set of obedience constraints for $t = 1, 2$ are similar to that of the static problem and are given by (18) and (19). Therefore, when the agents observe the traffic at $t = 1$, the problem of designing an optimal



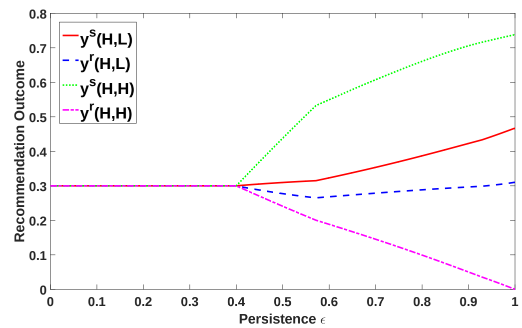
(a) The social welfare



(b) The number of agents at routes at $t = 1, 2$ for different route r's condition



(c) The recommendation outcome at $t = 2$ for $\theta_1 = L$



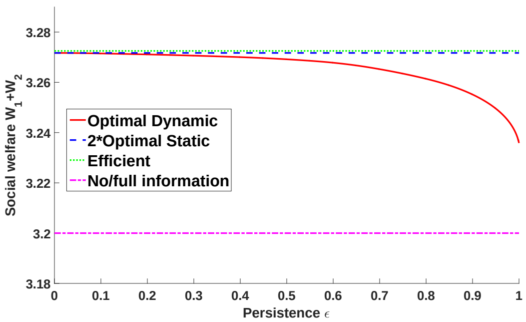
(d) The recommendation outcome at $t = 2$ for $\theta_1 = H$

Fig. 7: Case (ii) - the optimal dynamic recommendation policy for different values of persistence ϵ, for $\theta \in \{L, H\}$, $L = 1.2$, $H = 2.8$, $p_L = p_H = 0.5$ ($\Delta = 0$, $\sigma^2 = 0.64$)

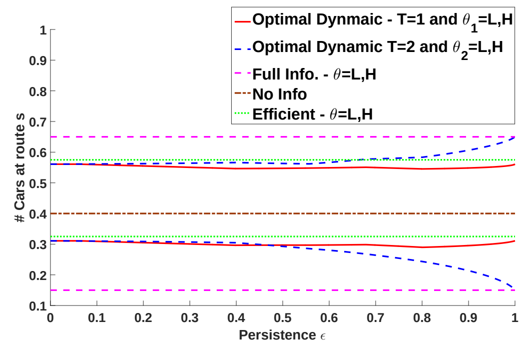
dynamic recommendation policy does not introduce any conceptual issue in addition to those present in the study of an optimal static recommendation policy. In the following, we argue that as the correlation between θ_1 and θ_2 increases the performance of the dynamic optimal recommendation policy decreases. Moreover, we show that when the correlation between θ_1 and θ_2 is high the designer's optimal recommendation policy at $t = 1$ is a partial information disclosure mechanism.[5]

We consider two classes of recommendation policies for the designer: (a) *dynamic separating recommendation policies* in which for every $\hat{\theta}_1 \neq \tilde{\theta}_1$, $\hat{\theta}, \tilde{\theta} \in \Theta$, we have $y_1(\hat{\theta}_1) \neq y_1(\tilde{\theta}_1)$, and

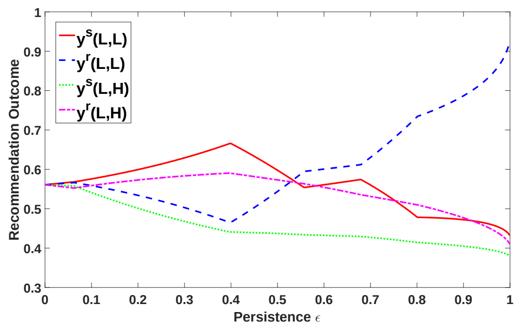
[5]For the sake of discussion we restrict attention to deterministic policies at $t = 1$. We can show that this restriction is without loss of optimality. For $t = 2$, fix the recommendation policy at $t = 1$. Then by an argument similar to the one given in the static setting in Section V, we can restrict without loss of optimality, to deterministic recommendation policies. For $t = 1$, fix the recommendation policy at $t = 2$, which is a deterministic recommendation policy. Then the problem of designing the optimal recommendation policy for $t = 1$ can be written as a linear program in terms of the probabilities of different routing policies for every realization of θ_1 (see [31] and [44]). It is known that in a linear program, the optimal solutions are the corner points. Therefore, under the optimal recommendation policy the probability of each routing policy is either 0 or 1. Therefore, we can restrict attention to the set of deterministic policies without loss of optimality.



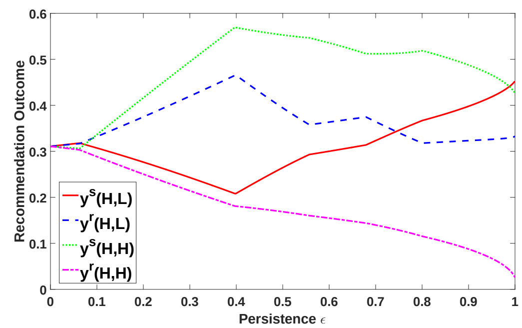
(a) The social welfare



(b) The number of agents at routes at $t = 1, 2$ for different route r's condition



(c) The recommendation outcome at $t = 2$ for $\theta_1 = L$



(d) The recommendation outcome at $t = 2$ for $\theta_1 = H$

Fig. 8: Case (iii) - the optimal dynamic recommendation policy for different values of persistence ϵ, for $\theta \in \{L, H\}$, $L = 1.7$, $H = 2.7$, $p_L = p_H = 0.5$ ($\Delta = 0.2$, $\sigma^2 = 0.25$)

(b) *dynamic pooling recommendation policies* in which there exists $\hat{\theta}_1 \neq \tilde{\theta}_1$, $\hat{\theta}, \tilde{\theta} \in \Theta$, we have $y_1(\hat{\theta}_1) = y_1(\tilde{\theta}_1)$.

First, consider a case where the principal employs a separating recommendation policy. As argued above, the agents can infer perfectly θ_1 at $t = 1$. Therefore, as ϵ increases the principal's information superiority is reduced at $t = 2$, and thus, the performance of the dynamic recommendation policy decreases at $t = 2$. Therefore, the overall performance of a dynamic separating recommendation policy is decreasing in ϵ. In particular, when $\theta_1 = \theta_2$ (*i.e.* perfect correlation), the agents learn θ_2 perfectly and the performance of the recommendation policy at $t = 2$ is the same as that of the full information disclosure mechanism.

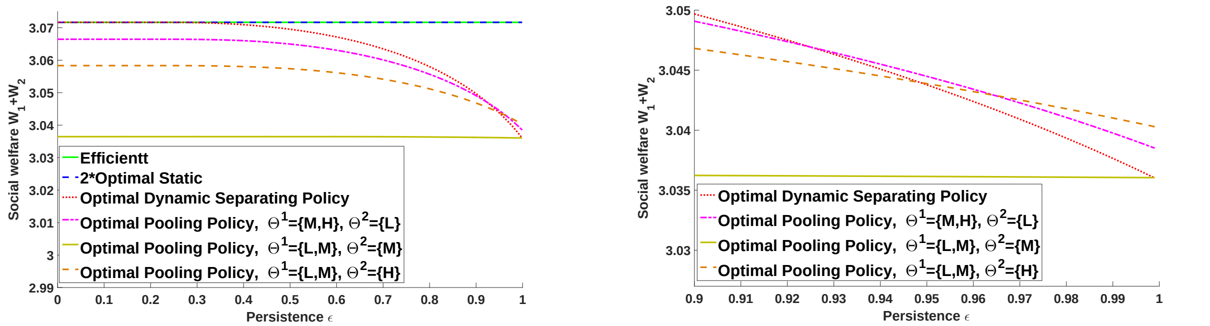
Next, consider a case where the principal employs a dynamic pooling recommendation policy. When the correlation between θ_1 and θ_2 is high, the principal may prefer to use a dynamic pooling recommendation policy so as to not reveal θ_1 perfectly at $t = 1$, and consequently, increase his information superiority at $t = 2$ as compared to the outcome under a dynamic separating recommendation policy. Given a dynamic pooling recommendation policy, let $\{\Theta^1, \Theta^2, \dots \Theta^m\}$ denote a partition of Θ such that for every $\hat{\theta}_1, \tilde{\theta}_1 \in \Theta$, we have $y_1(\hat{\theta}_1) = y_2(\tilde{\theta}_1)$ if and only if $\hat{\theta}_1, \tilde{\theta}_1 \in \Theta^i$ for some $1 \le i \le M$. That is, if Θ^i, $1 \le i \le m$, is a singleton, the principal implements a distinct routing outcome at $t = 1$ and the drivers perfectly learn the realization of θ_1 at $t = 1$; if Θ^i, $1 \le i \le m$, is not a singleton, the principal implements the same routing outcome for all realization in Θ^i and the drivers only learn that $\theta_1 \in \Theta^i$ at $t = 1$. Similar to the outcome under a dynamic separating recommendation policy, under a dynamic pooling recommendation policy the principal's information superiority decreases at $t = 2$ as the correlation between θ_1 and θ_2 increases since the drivers learn the partition to which θ_1 belongs. Therefore, the overall performance of the principal's optimal dynamic recommendation policy decreases as the correlation between θ_1 and θ_2 increases irrespective of the exact form of the optimal dynamic recommendation policy.

It is clear that the performance of an optimal dynamic pooling recommendation policy at $t = 1$ is inferior to that of an optimal dynamic separating recommendation policy. However,the performance of an optimal dynamic pooling recommendation policy at $t = 2$ is higher than that of an optimal dynamic separating recommendation policy since the principal has a higher information superiority under an optimal dynamic pooling recommendation policy rather than the one under an optimal dynamic separating recommendation policy. Using a numerical simulation, we show below that, when the correlation between θ_1 and θ_2 is high, there are instances where the principal's optimal recommendation policy is a dynamic pooling recommendation policy.

Consider a setting where $\Theta = \{L, M, H\}$ with $p_L = p_M = p_H = \frac{1}{3}$. Assume that the transition matrix $\mathbf{P}$ is given by

$$\mathbf{P} := \begin{pmatrix} p_L + \frac{2\epsilon}{3} & p_M - \frac{\epsilon}{3} & p_H - \frac{\epsilon}{3} \\ p_L - \frac{\epsilon}{3} & p_M + \frac{2\epsilon}{3} & p_H - \frac{\epsilon}{3} \\ p_L - \frac{\epsilon}{3} & p_M - \frac{\epsilon}{3} & p_H + \frac{2\epsilon}{3} \end{pmatrix}, \tag{38}$$

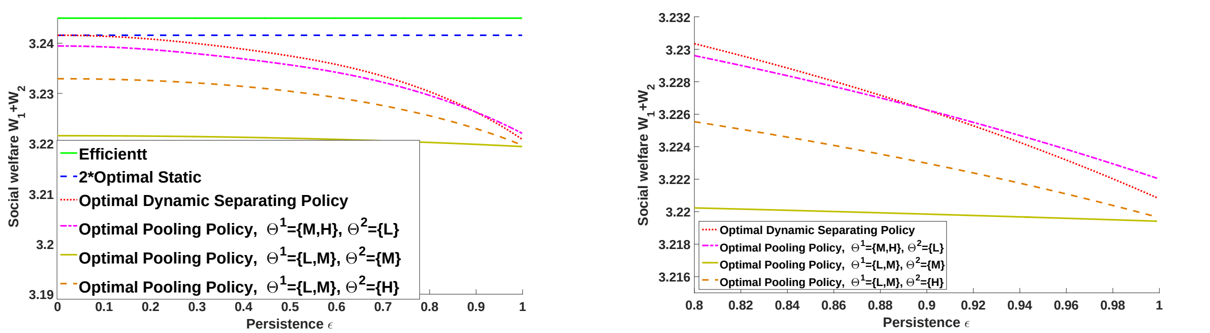
where $\epsilon \in [0, 1]$ denotes the persistence (i.e. correlation) of route r's condition over time. Figures 9 and 10 depict the optimal dynamic recommendation policy for two pairs of σ^2 and Δ. In the first example (see Figure 9), the parameters σ^2 and Δ satisfy the condition (22) of Theorem 2, while in the second example (see Figure 10) they do not satisfy it. As seen in Figures 9a and 9a, the performance of an optimal dynamic pooling recommendation policy and an optimal dynamic separating recommendation policy are decreasing in persistence ϵ. Figures 9b and 9b depict the optimal dynamic recommendation policies for high values of ϵ. As we discussed above, when the correlation between θ_1 and θ_2 is high, the principal prefers to employ a dynamic pooling recommendation policy.



(a) The social welfare under the optimal dynamic recommendation policy for different values of persistence ϵ

(b) The optimal recommendation policy for high values of ϵ

Fig. 9: Case (iii) - The optimal dynamic recommendation policy for $\{L, M, H\} = \{1.3, 2.1, 2.6\}$ ($\Delta = 0$, $\sigma^2 = 0.2867$)



(a) The social welfare under the optimal dynamic recommendation policy for different values of persistence ϵ

(b) The optimal recommendation policy for high values of ϵ

Fig. 10: Case(iii) - The optimal dynamic recommendation policy for different values of persistence ϵ, for $\{L, M, H\} = \{1.7, 2.3, 2.6\}$ ($\Delta = 0.2$, $\sigma^2 = 0.0867$)

VII. Conclusion

We investigated the problem of information disclosure mechanisms design in transportation networks. We showed that the principal can improve the social welfare by strategically disclosing information to the drivers, and coordinating the routing recommendations she provides to them. We characterized a condition under which the principal can implement the efficient routing outcome by utilizing her superior information to provide informational incentives to the drivers. We also investigated a two-time step dynamic setting where the drivers learn from their experience at $t = 1$. We characterized different pieces of information from which the drivers can learn, and examined the effect of each of them using numerical simulations. For future research, we will investigate the dynamic setting more extensively and consider the extension of our results for nonlinear congestion cost functions.

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